

Name: Key

Date: \_\_\_\_\_ Class: \_\_\_\_\_

5.4 Multiple Angle Identities Practice Worksheet

1. Simplify:  $\frac{\cos 2x}{\sin x + \cos x}$

$$\frac{\cos^2 x - \sin^2 x}{\sin x + \cos x}$$

$$\frac{(\cos x - \sin x)(\cancel{\cos x + \sin x})}{\cancel{\sin x + \cos x}}$$

$$= \boxed{\cos x - \sin x}$$

2.  $\sin 2x - \cos x = 0$

$$2 \sin x \cos x - \cos x = 0$$

$$\cos x (2 \sin x - 1) = 0$$

$$\cos x = 0 \quad \sin x = \frac{1}{2}$$

$$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}}$$

3.  $\tan 2x + \tan x = 0$

$$\frac{2 \tan x}{1 - \tan^2 x} + \tan x = 0$$

$$2 \tan x + \tan x - \tan^3 x = 0$$

$$3 \tan x - \tan^3 x = 0$$

$$\tan x (3 - \tan^2 x) = 0$$

$$\tan x = 0 \quad \sqrt{\tan^2 x} = \sqrt{3}$$

$$\tan x = \pm \sqrt{3}$$

$$\boxed{x = 0, \pi, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$$

4.  $\cos 2x = 3 \sin x + 2$

$$1 - 2 \sin^2 x = 3 \sin x + 2$$

$$2 \sin^2 x + 3 \sin x + 1 = 0$$

$$2x^2 + 3x + 1 = 0$$

$$(2x + 1)(x + 1) = 0$$

$$\sin x = -\frac{1}{2} \quad \sin x = -1$$

$$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{3\pi}{2}}$$

5.  $\sin 9x + \sin 18x = 0$       $\theta = 9x$

$$\sin \theta + \sin 2\theta = 0$$

$$\sin \theta + 2 \sin \theta \cos \theta = 0$$

$$\sin \theta (1 + 2 \cos \theta) = 0$$

$$\sin \theta = 0 \quad \cos \theta = -\frac{1}{2}$$

$$\theta = 0, \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$\boxed{x = 0, \frac{\pi}{9}, \frac{2\pi}{27}, \frac{4\pi}{27}}$$

6. Prove:  $2 \sin x \cos^3 x + 2 \sin^3 x \cos x = \sin 2x$

$$2 \sin x \cos x (\cancel{\cos^2 x} + \cancel{\sin^2 x})$$

$$2 \sin x \cos x = \boxed{\sin 2x}$$

✓

Angle Identities

7. Find the exact value of  $\sin \frac{5\pi}{8}$  use  $\frac{5\pi}{4}$

$$\sin \frac{\theta}{2} = (+) \sqrt{\frac{1 - \cos \theta}{2}} = \sqrt{\frac{1 - \cos \frac{5\pi}{4}}{2}}$$

$$\sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2 + \sqrt{2}}{2}}$$

$$= \sqrt{\frac{2 + \sqrt{2}}{4}} = \boxed{\frac{\sqrt{2 + \sqrt{2}}}{2}}$$

8. Find the exact value of  $\cos \frac{\pi}{12}$  use  $\frac{\pi}{6}$

$$\cos \frac{\theta}{2} = (+) \sqrt{\frac{1 + \cos \theta}{2}} = \sqrt{\frac{1 + \cos \frac{\pi}{6}}{2}}$$

$$= \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2 + \sqrt{3}}{2}}$$

$$= \sqrt{\frac{2 + \sqrt{3}}{4}} = \boxed{\frac{\sqrt{2 + \sqrt{3}}}{2}}$$

9. Find the exact value of  $\tan \frac{11\pi}{12}$  use  $\frac{11\pi}{6}$

$$\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta} = \frac{1 - \cos \frac{11\pi}{6}}{\sin \frac{11\pi}{6}}$$

$$\frac{1 - \frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \frac{2 - \sqrt{3}}{2} = \frac{2 - \sqrt{3}}{-\frac{1}{2}}$$

$$= \boxed{-2 + \sqrt{3}}$$

10.  $\sin^2\left(\frac{x}{2}\right) = 2\cos^2x - 1$

(Hint: Use half angle identities)

$$\left(\sqrt{\frac{1 - \cos x}{2}}\right)^2 = 2\cos^2x - 1$$

$$\frac{1 - \cos x}{2} = 2\cos^2x - 1$$

$$1 - \cos x = 4\cos^2x - 2$$

$$4\cos^2x + \cos x - 3 = 0$$

$$(4x - 3)(x + 1)$$

$$\cancel{\cos x = \frac{3}{4}} \quad \cos x = -1 \quad \boxed{x = \pi}$$