

5.1 Practice: Simplifying and Solving Trig Expressions and Equations

Part 1: Simplifying Trig

1. $\cos \theta + \tan \theta \sin \theta$

$$\cos \theta + \frac{\sin \theta}{\cos \theta} \cdot \sin \theta$$

$$\cos \theta + \frac{\sin^2 \theta}{\cos \theta}$$

$$\frac{\cos^2 \theta + \sin^2 \theta}{\cos \theta} = \frac{1}{\cos \theta}$$

$$= \boxed{\sec \theta}$$

2. $\frac{1+\sin \theta}{\cos \theta} + \frac{\cos \theta}{1+\sin \theta}$

$$\frac{1+2\sin \theta + \sin^2 \theta + \cos^2 \theta}{\cos \theta (1+\sin \theta)}$$

$$\frac{2+2\sin \theta}{\cos \theta (1+\sin \theta)} \rightarrow \frac{2(1+\sin \theta)}{\cos \theta (1+\sin \theta)}$$

$$\frac{2}{\cos \theta} = \boxed{2 \sec \theta}$$

3. $\frac{\tan \theta + \cot \theta}{\sin \theta \cos \theta}$

$$\frac{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}}{\sin \theta \cos \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}$$

$$\frac{1}{\sin \theta \cos \theta} \cdot \frac{1}{\sin \theta \cos \theta} =$$

$$\frac{1}{\sin^2 \theta \cos^2 \theta} = \boxed{\csc^2 \theta \sec^2 \theta}$$

4. $\frac{1}{1-\sin \theta} + \frac{1}{1+\sin \theta}$

$$\frac{1+\sin \theta + 1-\sin \theta}{(1-\sin \theta)(1+\sin \theta)}$$

$$\frac{2}{1-\sin^2 \theta} = \frac{2}{\cos^2 \theta}$$

$$= \boxed{2 \sec^2 \theta}$$

5. $\frac{\sin \theta}{\cos \theta + 1} + \frac{\cos \theta - 1}{\sin \theta}$

$$\frac{\sin^2 \theta + \cos^2 \theta - 1}{(\cos \theta + 1) \sin \theta}$$

$$\frac{1-1}{(\cos \theta + 1) \sin \theta} = \frac{0}{(\cos \theta + 1) \sin \theta}$$

$$= \boxed{0}$$

6. $\frac{\cos^2 \theta - \sin^2 \theta}{1 - \tan^2 \theta}$

$$\frac{\cos^2 \theta - \sin^2 \theta}{1 - \frac{\sin^2 \theta}{\cos^2 \theta}} = \frac{\cos^2 \theta - \sin^2 \theta}{\frac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta}}$$

$$\frac{(\cos^2 \theta - \sin^2 \theta) \cdot \cos^2 \theta}{\cos^2 \theta - \sin^2 \theta}$$

$$= \boxed{\cos^2 \theta}$$

7. $3 \sec^2 x - 5 \sec x - 2 = 0$

$$3x^2 - 5x - 2 = 0$$

$$(3x + 1)(x - 2) = 0$$

$$\sec x = -\frac{1}{3} \quad \sec x = 2$$

$$\cancel{\cos x = -3} \quad \cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

8. $2 \sin^2 x - \cos x = 1$

$$2(1 - \cos^2 x) - \cos x - 1 = 0$$

$$2 - 2\cos^2 x - \cos x - 1 = 0$$

$$0 = 2\cos^2 x + \cos x - 1 = 0$$

$$2x^2 + x - 1 = 0$$

$$(2x - 1)(x + 1) = 0$$

$$\cos x = \frac{1}{2} \quad \cos x = -1$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$$

9. $\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = 4$

$$\frac{1 + 2\sin x + \sin^2 x + \cos^2 x}{\cos x(1 + \sin x)} = 4$$

$$\frac{2 + 2\sin x}{\cos x(1 + \sin x)} = 4 \rightarrow \frac{2(1 + \sin x)}{\cancel{\cos x(1 + \sin x)}}$$

$$\frac{2}{\cos x} = 4 \rightarrow 2 = 4\cos x$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

10. $\csc^2 x - 3\csc x + 2 = 0$

$$x^2 - 3x + 2 = 0$$

$$(x - 2)(x - 1) = 0$$

$$\csc x = 2 \quad \csc x = 1$$

$$\sin x = \frac{1}{2} \quad \sin x = 1$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$$