A New Way to Look at Mathematics

On the pages that follow, you will be introduced to a wide range of real world problems that operations researchers solve every day. While the real world problems can be difficult to solve due to their enormous size, in many cases, their solution depends on mathematics that you have already learned in your previous high school mathematics courses. So how, you may wonder, will this course be different?

This new course in the mathematics of operations research for high school students is different because it is applications-based and problem-driven. This means that the applications of the mathematics will be upfront, not at the end of the chapter and that the key ideas will be developed within the sorts of problems that gave them birth. The applications you will see focus around making decisions in business, government, or your own personal life.

In addition, mathematics is not a spectator sport. Therefore, in the text, many questions are asked but not answered. Instead, you will provide the answers. Studying mathematics in this way may require you to develop a new mindset about what mathematics is, how it can be used, and the best way to learn it.

Section 1.0: Introduction to Making Hard Decisions

We all face decisions in our jobs, in our communities, and in our personal lives. For example,

- Where should a new airport, manufacturing plant, power plant, or health care clinic be located?
- Which college should I attend, or which job should I accept?
- Which car, house, computer, stereo, or health insurance plan should I buy?
- Which supplier or building contractor should I hire?

Decisions such as these involve comparing alternatives that have strengths or weaknesses with regard to multiple objectives of interest to the decision. For example, your criteria in buying health insurance might be to minimize cost *and* maximize protection. Sometimes these multiple criteria get in each other's way. *Multi-criteria decision making* (MCDM) is used when one needs to make a hard decision with many criteria. In this chapter, you will see one form of multi-criteria decision making. The method introduced in this chapter is a structured methodology designed to handle the tradeoffs among multiple criteria.

A Little History

One of the first applications of this method of MCDM involved the study of possible locations for a new airport in Mexico City in the early 1970s. The criteria considered included cost, capacity, access time to the airport, safety, social disruption, and noise pollution.

The problems in this chapter use the steps of multi-criteria decision making to make hard decisions. MCDM is a systematic approach to quantify an individual's preferences. Measures of interest are rescaled to numerical values on a 0–1 scale, with 0 representing the worst value of the measure and 1 representing the best. This allows the direct comparison of many diverse measures. In other words, with the right tool, it really is possible to compare apples to oranges! The result of this process is an evaluation of the alternatives in a rank order that reflects the decision makers' preferences.

For example, individuals, college sports teams, Master's degree programs, or even hospitals can be ranked in terms of their performance on many diverse measures. Another example is the Bowl Championship Series (BCS) in college football that attempts to identify the two best college football teams in the United States to play in a national championship bowl game. This process has reduced, but not eliminated, the annual end-of-year arguments as to which college should be crowned national champion.

Section 1.1: Choosing a Cell Phone Plan

Choosing a cell phone plan is an important decision for many people. In fact, approximately 75% of all teenagers own cell phones. When choosing a cell phone plan, there are many factors to consider.

Q1. What factors would you consider if you were choosing a cell phone plan?

In this chapter, you will develop a process for making large decisions such as choosing a cell phone plan. Before doing so, you will complete an opening activity.

1.1.1 Opening Activity

In this opening activity, you will make a decision about what cell phone plan you would choose if you were considering a new plan. To do so, complete the following steps:

1. Make a list of possible cell phone plans that you would consider using.
2. Collect data on each of these plans that you would find useful in making a decision.
3. Choose one of the plans based on your data.
4. Explain why you chose this plan over the others.

Q2. What possible issues do you foresee with using these steps to choose a cell phone plan?

In the following sections, the steps of the MCDM process will be explained in the context of a high school student and her friend helping her parents to choose a cell phone plan. Isabelle Nueva needs to help her mother and father decide on the best cell phone plan for her family. She and her friend, Angelo Franco, will use the MCDM process they learned in their math class to help her parents make this decision. Follow along with Isabelle as she and Angelo use the MCDM process to make this decision.

1.1.2 Identify Criteria and Measures

The first thing they do is identify the **criteria** of a cell phone plan that were important to Isabelle's family. From discussions she had with her mother and father, Isabelle knew that the criteria that were important to them were cost, contract features, and phone features.

Q3. If you were choosing a cell phone plan, what criteria would be important to you?

Isabelle and Angelo know that they need to find at least one way to **measure** each of the criteria. They decide to measure the Cost criterion using Monthly Service Charge. They decide to measure the Contract Features criterion using Number of Minutes per Month, Minimum Contract Length, whether there is Free Unlimited Texting, and whether there is an Included Data Plan. They measure the Phone Features using Quality of Service. Each criterion and its measures are provided in Table 1.1.1.

Criteria	Measures
Cost	Monthly Service Charge
Contract Features	Number of Minutes per Month
	Minimum Contract Length
	Free Unlimited Texting
	Included Data Plan
Phone Features	Quality of Service

Table 1.1.1: Criteria and measures for choosing a cell phone plan

Q4. How would you measure each of your criteria?

The value of two of the measures—the Monthly Service Charge and the Number of Minutes per Month—could be any numerical amount within a reasonable range. These are examples of **continuous measures**. That is, these measures can take on any numerical value within a range.

Isabelle and Angelo decide that the data they collected for the other three measures can be grouped into a finite number of categories. For example, Free Unlimited Texting has only two categories: "yes" and "no." Similarly, Included Data Plan has two categories: "yes" and "no." Thus, these two measures are examples of **categorical measures**.

Q5. Of the measures you listed in Q4, which are continuous and which are categorical?

The next categorical measure Isabelle and Angelo consider is Quality of Service. For this measure, they decide to use ratings from a consumer magazine. The magazine considered dropped or disconnected calls, static and interference, and voice distortion to rate the quality of service. Isabelle and Angelo decide to only consider plans the magazine rated "Good", "Very Good", or "Excellent". Therefore, this measure has three categories.

The last categorical measure is Minimum Contract Length—the shortest time a customer must remain with a particular plan to avoid paying a fee to cancel the service. Isabelle's parents were concerned about being locked into a plan for a long period of time. The plans under consideration have three different minimum contract lengths (6 months, 1 year, and 2 years). Thus, the Minimum Contract Length measure has three categories.

The categorical measures and their possible values are provided in Table 1.1.2.

Categorical Measure	Category
Free Unlimited Texting	Yes No
Included Data Plan	Yes No
Quality of Service	Excellent Very Good Good
Minimum Contract Length	6 months 1 year 2 years

Table 1.1.2: Categorical variables with categories and numeric values

Q6. Create a table similar to Table 1.1.2 for your categorical measures identified in the previous question. In order to do this, you will need to research possible cell phone plans. What sort of research would you need to do?

1.1.3 Collect Data

Isabelle's parents were considering three cell phone plans: Trot, UST&T, and Horizon. Isabelle and Angelo collected the data they need to help her parents make their decision. The data appear in Table 1.1.3.

Plan	Monthly Charge ($)	Minutes per Month	Minimum Contract Length	Free Unlimited Texting	Included Data Plan	Quality of Service
Trot	60	400	6 months	no	yes	Good
UST&T	75	500	2 years	yes	no	Excellent
Horizon	85	600	1 year	yes	no	Very Good

Table 1.1.3: Isabelle and Angelo's cell phone data

Q7. Create a table similar to Table 1.1.3 for your cell phone data.

1.1.4 Find the Range of Each Measure

Next, Isabelle and Angelo specify a range for each measure. They first specify the range for the two continuous measures (Monthly Service Charge and Minutes per Month). For each of these measures, they decide to use the range of the data they collected. That is, for monthly service charge, the range was 60-85, and the range for minutes per month was 400-600.

For the categorical measures, Isabelle and Angelo assign numerical values for each category. When there were only two categories, the more desirable category was assigned a "1" and the less desirable category was assigned a "2"., When there were three categories, the most desirable category was assigned a "1", the middle category was assigned a "2", and the least desirable category was assigned a "3".

The **scale ranges** for each of Isabelle and Angelo's measures are given in Table 1.1.4.

Measure	Scale range
Monthly Charge	$60-$85
Minutes per Month	400-600 minutes
Minimum Contract Length	6 months
	1 year
	2 years
Free Unlimited Texting	Yes
	No
Included Data Plan	Yes
	No
Quality of Service	Excellent
	Very Good
	Good

Table 1.1.4: Ranges of each measure

Q8. Specify the ranges for each of your measures, and create a table similar to Table 1.1.4.

1.1.5 Rescale Each Measure to a Common Unit

It would be difficult to compare the three plans using this raw data. For example, how would one compare a $10 difference in the monthly service charge to a 1 year difference in minimum contract length? In order to avoid such problems, operations researchers rescale the range of each measure to common unit values between 0 and 1. This creates for each measure a **common unit** that varies from 0 to 1, where 0 always represents the worst value and 1 the best value.

Converting Continuous Measures
For both of the continuous measures, Isabelle and Angelo use a **proportional scale**. For example, the range for the Monthly Service Charge measure is from $60 to $85. Since the smallest possible value here is the best option and since 1 represents the best option, $60 is converted to a common unit value of 1. Similarly, the largest possible value of the monthly service charge is the worst option, so $85 is converted to 0. That is,

$$\$60 \rightarrow 1$$
$$\$85 \rightarrow 0$$

Next, Isabelle and Angelo convert the price of UST&T's plan to a common unit value. They must decide what $75 should be converted to when it was compared to the best and worst options ($65 and $85, respectively) for monthly service charge. The graph in Figure 1.1.1 illustrates this.

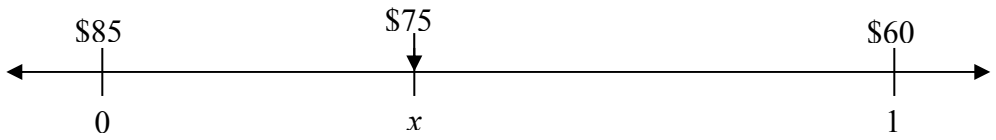


Figure 1.1.1: Determining the common unit values for the monthly service charge measure

Q9. Is $75 closer to the best or the worst option?

Q10. How far is $75 from the best option? How far from the worst?

Q11. Using your responses to the previous questions, what do you think $75 should be converted to?

Isabelle and Angelo solve a proportion to arrive at the common unit value for the Monthly Service Charge of $75. To find the common unit value for $75 using proportions, Isabelle and Angelo write two equivalent fractions in the form $\frac{\text{part}}{\text{whole}}$. Figure 1.1.2 illustrates this.

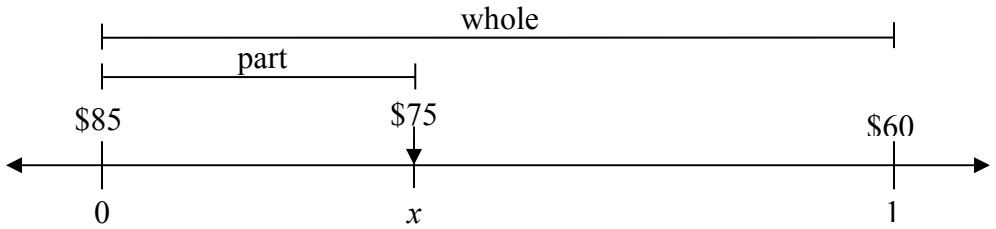


Figure 1.1.2: Determining the proportion to find the common unit values

In the first fraction, the "part" refers to the distance between \$85 and \$75, and the "whole" refers to the distance between \$85 and \$60. In the second fraction, the "part" refers to the distance between 0 and *x*, and the "whole" refers to the distance between 0 and 1. As can be seen in Figure 1.1.2, these two fractions are equivalent.

Isabelle and Angelo solve for the unknown in the equivalent fractions, using absolute value to find the distance between two values.

$$\frac{|75-85|}{|60-85|} = \frac{|x-0|}{|1-0|}$$

$$\frac{10}{25} = \frac{x}{1}$$

$$0.4 = x$$

Therefore, \$75 is converted to 0.4.

Notice, each time these equivalent fractions are developed, the fraction on the right will always be:

$$\frac{|x-0|}{|1-0|} = \frac{x}{1} = x$$

Therefore, there is no need to write the entire fraction. Simply *x* can be used instead.

Q12. What other ways could Isabelle and Angelo use to calculate the common unit value for \$75?

Q13. Find the common unit values for the Minutes per Month measure.

Q14. Find the common unit values for each of your continuous measures from Q5.

Converting Categorical Measures

For the four categorical measures, Isabelle and Angelo assigned a common unit value of 0 to the worst option and 1 to the best option.

When there were only two (e.g., the Free Unlimited Texting and the Included Data Plan measures), the best option was assigned 1 and the worst option was assigned 0. When there was something between the best and worst values (e.g., the Minimum Contract Length and Quality of Service measures), the value in the middle was assigned a common unit value of 0.5. This is shown in Table 1.1.5.

Categorical Measure	Scale range	Common Units
Minimum Contract Length	6 months	1
	1 year	0.5
	2 years	0
Free Unlimited Texting	Yes	1
	No	0
Included Data Plan	Yes	1
	No	0
Quality of Service	Excellent	1
	Very Good	0.5
	Good	0

Table 1.1.5: Common units for the categorical measures

Q15. Find the common unit values for each of your categorical measures from Q5.

Table 1.1.6 contains all of the common unit values for each of the six measures for each of the three plans.

Plan	Monthly Charge ($)	Minutes per Month	Minimum Contract Length	Free Unlimited Texting	Included Data Plan	Quality of Service
Trot	1	0	1	0	1	0
UST&T	0.4	0.5	0	1	0	1
Horizon	0	1	0.5	1	0	0.5

Table 1.1.6: Cell phone data converted to a common unit

When Isabelle and Angelo looked at these results, they noticed that each plan received the top common unit value of 1 on at least two of the measures. They also noticed that each plan received at least one bottom common unit value of 0. Therefore, it is not obvious to them which plan they should choose.

Q16. Based on the common unit values, which plan should Isabelle recommend to her parents?

Angelo thinks they should use the total of all of the common units to get a total score for each plan. Using his system, Angelo gets the following scores:

Plan	Total Score
Trot	3
UST&T	2.9
Horizon	3

Table 1.1.7: Total scores for each plan

Isabelle thinks it will be more meaningful to compute the average common unit scores for each plan. To do so, she divided the total score for each plan by six (the total possible score). She obtained the following approximate averages:

Plan	Average Score
Trot	0.5
UST&T	0.48
Horizon	0.5

Table 1.1.8: Average scores for each plan

Q17. Do you think it makes more sense to use the sum or the average to make a decision?

Q18. Based on the total and average scores, which plan do you think Isabelle should recommend to her parents? Why?

Q19. What are some reasons why Isabelle may not recommend Horizon to her parents?

Q20. What are some reasons why Isabelle may think Trot would be a better choice for her parents?

Q21. What are some reasons why Isabelle may think UST&T would be a better choice for her parents?

Q22. Calculate the total scores and the average scores for each of your cell phone plans.
   a. Based on these values, which plan would you choose?
   b. What are some reasons why these plans may not be the best choice for you?
   c. Was this plan what you expected to choose based on the opening activity? Why or why not?

Whether they use the sum or the average, Isabelle and Angelo realize that each plan has something in its favor. They wonder how to reach a decision. Then Isabelle remembers that her parents were really worried about the monthly service charge, and not as worried about the length of the contract. They decide that they need a system that does not treat all of the measures equally, as the sum and average do. They need a system that weights each measure according to how important it is to Isabelle's parents.

1.1.6 Conduct an Interview to Calculate Weights

In order to learn how important each measure is to her parents, Isabelle and Angelo decide to interview them. They want to learn for which measure Isabelle's parents believe is most important to have the most preferred value rather than the least preferred value. To find out, they ask Isabelle's parents to rank the five measure ranges in their order of importance. They decided that the difference between the highest and lowest monthly payments was most important to them. They, therefore, assigned "monthly charge" a rank of 1. Because they were concerned about the possibility of running up huge bills for texting, they made free texting their second most important measure.

Table 1.1.9 shows their rank-ordering of the measures. For example, monthly service charge is the most important measure to Isabelle's parents and minimum contract length is the least important. This table also includes the least preferred value and the most preferred value for each of the measures.

Measure	Monthly Charge ($)	Minutes per Month	Minimum Contract Length	Free Unlimited Texting	Included Data Plan	Quality of Service
Least Preferred Value	85	400	2 years	No	No	Good
Most Preferred Value	60	600	6 months	Yes	Yes	Excellent
Rank	1	4	6	2	5	3

Table 1.1.9: Rank-order of the measures according to Isabelle's parents

Q23. Rank-order each of your measures.

Next, Isabelle and Angelo assign weights to each measure that capture more than the order of importance. They need a sense of how much more important one measure is than another. For example, if one measure is twice as important as another, then the assigned weights should reflect the strength of that difference.

Isabelle and Angelo ask Mr. and Mrs. Nueva to assign 100 points to the measure they ranked number 1—the measure they consider most important. Then, they ask them to assign a number of points less than 100 to the second-ranked measure, free unlimited texting. In doing so, they ask Isabelle's parents to pick a number that reflects how important it is compared to the number one ranked measure. Mr. and Mrs. Nueva choose to assign 90 points to free unlimited texting, because they know Isabelle likes to text a lot.

The interview continues until points have been assigned to each of the six measures. Table 1.1.10 shows the points that Mr. and Mrs. Nueva assigned to each measure.

Measure	Monthly Charge ($)	Minutes per Month	Minimum Contract Length	Free Unlimited Texting	Included Data Plan	Quality of Service
Least Preferred Value	85	400	2 years	No	No	Good
Most Preferred Value	60	600	6 months	Yes	Yes	Excellent
Rank	1	4	6	2	5	3
Points	100	75	50	90	60	80

Table 1.1.10: Points assigned to each of the measures

Q24. Assign points to each of your measures, and create a table similar to Table 1.1.10.

Now, Isabelle and Angelo total all of the assigned points and obtain 455. Then, they divide the point assignment for each measure by that total. This number is the **weighted score** (or just "weight") of that measure. For example, monthly charge was assigned 100 points. Thus, the weight of this measure is:

$$\frac{100}{455} = 0.22$$

Table 1.1.11 shows the calculated weight for each of the other measures.

Measure	Monthly Charge ($)	Minutes per Month	Minimum Contract Length	Free Unlimited Texting	Included Data Plan	Quality of Service	Total
Rank	1	4	6	2	5	3	--
Points	100	75	50	90	60	80	455
Weight	$^{100}/_{455} = 0.22$	0.16	0.11	0.20	0.13	0.18	1.00

Table 1.1.11: A weight is calculated for each measure

Q25. What is the largest weight? Which is the smallest?

Q26. What is the ratio of the largest weight to the smallest weight?

Q27. What should this ratio mean in the context of the decision?

Q28. Assign points to each of your measures, and create a table similar to Table 1.1.11.

1.1.7 Calculate Total Scores

Now, Isabelle and Angelo calculate a **total score** for each plan. The total score is an example of a weighted average. They multiply each common unit value from Table 1.1.6 by the corresponding weight from Table 1.1.11. Then for each plan, they add those six products together to get the total score.

Table 1.1.12 shows the results of these computations. Notice that this weighted average captures how important the various measures are to Isabelle's parents. Notice also that on the basis of this weighted average approach, UST&T is the preferred choice by Isabelle's parents.

Plan	Monthly Charge ($)	Minutes per Month	Minimum Contract Length	Free Unlimited Texting	Included Data Plan	Quality of Service	Total
Trot	(0.22)(1) = 0.22	(0.16)(0) = 0	(0.11)(1) = 0.11	(0.20)(0) = 0	(0.13)(1) = 0.13	(0.18)(0) = 0	0.46
UST&T	0.088	0.08	0	0.20	0	0.18	0.548
Horizon	0	0.16	0.055	0.20	0	0.09	0.505

Table 1.1.12: A weighted total score is computed for each plan.

Q29. Multiply the common unit values by the corresponding weights for each of your plans, and create a table similar to Table 1.1.12.

Q30. Would everyone's score results lead to the same preferred choice? Explain.

1.1.8 Determine Strengths/Weaknesses and Make Final Decision

Isabelle and Angelo decide to examine their results because the total scores of UST&T and Horizon are so close. They are also concerned because their weighting system does not produce the same result as the sum or average methods.

Q31. For which measures does UST&T have a higher weighted score than Horizon? For which does Horizon outscore UST&T?

Q32. What are the ranks of the measures where UST&T scored higher than Horizon (see Table 1.1.9)? What are the ranks for the measures where Horizon was higher?

When Isabelle and Angelo compare UST&T with Horizon, they see that UST&T had higher weighted scores for the first and third ranked measures. Horizon scored higher on the fourth and sixth ranked measures. UST&T and Horizon were tied on the second and fifth ranked measures. UST&T scored better on two of the three most important measures. In contrast, Horizon scored better only on the two of the least important measures. Therefore, Isabelle and Angelo believe that their weighting system did what it was supposed to do; it took account of Mr. & Mrs. Nueva's preferences. They decide to recommend the UST&T plan to Isabelle's parents.

1.1.9 Summary

In this problem, Isabelle and Angelo needed to choose a cell phone plan for Isabelle's parents. They completed the following steps:

1. Identify Criteria and Measures
2. Collect Data
3. Find the Range of Each Measure
4. Rescale Each Measure to a Common Unit

After completing these steps, Isabelle and Angelo found the total score and the average score for each cell phone plan. However, they found that these values took all measures into consideration equally. This was not a reasonable way to make a decision. They needed a way to weigh some measures more than others, because Isabelle's parents were more concerned about the cost of the plan than anything else.

In order to take account of Mr. and Mrs. Nueva's preferences regarding a cell phone plan, Isabelle and Angelo completed three additional steps:

5. Conduct an Interview to Calculate Weights
6. Calculate a Total Score for Each Alternative
7. Interpret Results.

This seven-step process will be applied in the next two sections and in the homework to make slightly more complicated decisions. This process is also a life-skill, because you may find it useful to help you make some important decisions in your future.

Section 1.2: Enrique Ramirez Chooses a College

Enrique Ramirez has been accepted at four colleges: Canisius College in Buffalo, NY; Clark University in Worcester, MA; Drexel University in Philadelphia, PA; and Suffolk University in Boston, MA. Now he must decide which one to attend.

Enrique asks his friend Anna for help. Enrique and Anna realize that there are many different issues to consider when making this decision. They also realize that the issues of interest to Enrique and their relative importance are not the same as those for Anna.

To make this decision, Enrique, with the help of Anna, follows the steps of MCDM that were presented in the previous section. These steps are given in Table 1.2.1.

General Steps	Descriptions for this Particular Decision
1. Identify Criteria and Measures	First, generate a list containing general criteria that are important when choosing a college. These criteria will be broad in nature and will be based on objective and subjective goals. Next, specify at least one measure for each criterion.
2. Collect Data	For each college, collect the data for each measure.
3. Find the Range of Each Measure	Specify a reasonable scale for each measure.
4. Rescale Each Measure to a Common Unit	Rescale each measure to common units from 0 to 1, with 0 being the worst alternative and 1 being the best alternative.
5. Conduct an Interview to Calculate Weights	With the help of an interviewer, rank-order the measures, assign points from 0 to 100 to each measure, and calculate a proportional weight between 0 and 1 for each measure.
6. Calculate Total Scores	Calculate a total weighted score for each college. These weights will yield a ranking of the colleges, allowing you to identify the best option based on your preferences.
7. Interpret Results	Review the results to understand the strengths and weaknesses of your top alternatives before finalizing your decision.

Table 1.2.1: Steps of MCDM

1.2.1 Identify Criteria and Measures

With Anna's help, Enrique decides that academics, cost, location, and social life are the criteria most critical in his choice of a school. Next, Enrique and Anna take his list of four criteria and specify two or three measures for each criterion. His criteria and measures are given in Table 1.2.2.

Criteria	Measures
Academics	Average SAT Score (based on last year's freshman class)
	U.S. News & World Report Ranking
Cost	Room & Board (annual)
	Tuition (annual)
Location	Average Daily High Temperature
	Nearness to Home
Social Life	Athletics
	Reputation
	Size

Table 1.2.2: Enrique's criteria and measures

1.2.2 Collect Data

For each measure, Enrique and Anna collect data, which is listed in Table 1.2.3. Some of the measures are naturally categorical. For example, *U.S. News & World Report* ranks schools into four categories:

1. Nationally ranked
2. Regionally ranked
3. Regionally tier 3
4. Regionally tier 4

Enrique and Anna divide Athletics into three categories:

1. Division 1
2. Division 2
3. Division 3

Similarly, they divide Reputation into three categories:

1. Seriously academic
2. Balanced academics and social life
3. Party school

The data for the remaining measures are numerical values. Enrique and Anna are able to find the average values for SAT score and daily high temperature and the exact values for room and board cost, tuition cost, nearness to home, and size.

Measure	Canisius	Clark	Drexel	Suffolk
Average SAT Score	1590	1750	1700	1480
U.S. News & World Report Ranking	22nd (regional)	91st (national)	109th (national)	Tier 3 (regional)
Room & Board	$10,150	$8,850	$12,135	$11,960
Tuition	$28,157	$33,900	$30,470	$25,850
Average Daily High Temperature	56°	56°	64°	59°
Nearness to Home	297 mi	157 mi	81 mi	191 mi
Athletics	Division 1	Division 3	Division 1	Division 3
Reputation	Balanced	Seriously academic	Seriously academic	Balanced
Size	3,300 students	2,175 students	12,348 students	4,985 students

Table 1.2.3: Raw data for Enrique's four schools

1.2.3 Find the Range of Each Measure

Next, Enrique and Anna choose an appropriate scale for each of the nine measures. Some of the measures are continuous (e.g., SAT score), while others are categorical (e.g., athletics).

For the Nearness to Home measure, Enrique believes that exact mileage is not important, but rather broad ranges of mileage better represent his concerns. Therefore, Enrique and Anna convert this measure from continuous to categorical.

Q1. Looking at Table 1.2.4, what other measure was converted from continuous to categorical?

Enrique and Anna also realize that the range of each scale is important. For example, the theoretical range of the average combined SAT score is 600–2400, but in actuality, the range of the average combined SAT score at the colleges Enrique is considering is 1480–1750, which is a much narrower range. Enrique and Anna decide that it is much more realistic to use a range that is close to the actual range.

Q2. In the previous section, the ranges for the continuous measures were simply the ranges of the data collected. In this section, the ranges are expanded slightly. For example, instead of the SAT range staying as 1480-1750, Enrique and Anna choose the range 1400-1800. Why might one prefer to use the ranges of the data collected? Why might one prefer to round the ranges?

Q3. Looking at Table 1.2.4, for what other measures do Enrique and Anna create realistic ranges? Do you agree with their ranges? Why or why not?

The scale range and type of each measure are given in Table 1.2.4.

Measure	Scale range	Type
Average SAT Score	1400–1800	Continuous
U.S. News & World Report Ranking	Nationally Ranked	Categorical
	Regionally Ranked	
	Regionally Tier 3	
	Regionally Tier 4	
Room & Board	$8,000–$14,000	Continuous
Tuition	$25,000–$35,000	Continuous
Average Daily High Temperature	50˚–70˚F	Continuous
Nearness to Home	Within 1 hr. Drive (50-100 mi)	Categorical
	Within 4 hr. Drive (101–200 mi)	
	Within a Day's Drive (201–300 mi)	
Athletics	Division 1	Categorical
	Division 2	
	Division 3	
Reputation	Seriously Academic	Categorical
	Balanced Academics and Social Life	
	Party School	
Size	Under 3,000 students	Categorical
	3,001–6,000 students	
	6,001–12,000 students	
	Over 12,000 students	

Table 1.2.4: Types and ranges of measures

Before continuing, Enrique and Anna convert the values of the categorical measures into the numerical values based on the ranges of each measure. The converted data for the categorical measures are given in Table 1.2.5.

Measure	Canisius	Clark	Drexel	Suffolk
Average SAT Score	1590	1750	1700	1480
U.S. News & World Report Ranking	2	1	1	3
Room & Board	$10,150	$8,850	$12,135	$11,960
Tuition	$28,157	$33,900	$30,470	$25,850
Average Daily High Temperature	56˚	56˚	64˚	59˚
Nearness to Home	3	2	1	2
Athletics	1	3	1	3
Reputation	2	1	1	2
Size	2	1	4	2

Table 1.2.5: Converted categorical data for Enrique's four schools

1.2.4 Rescale Each Measure to a Common Unit

Once Enrique and Anna choose appropriate scales for each of the measures, Anna reminds Enrique that if they compared the data in its current form, it would be like comparing apples to oranges. They decide to convert the data to common units. To do so, they assign 1 to the best value and 0 to the worst value in the range of each measure. Recall from the previous section, the method for determining intermediate values differs for continuous and categorical measures.

Converting Continuous Measures
For the continuous measures, Enrique and Anna use a proportional scale. For example, the Average SAT Score at Canisius is 1590. The range for this measure is 1400–1800, so 1400 (the least desirable score) should be converted to 0 and 1800 (the most desirable score) to 1. But what should the proportional value for Canisius be?

$$1800 \rightarrow 1$$
$$1590 \rightarrow x$$
$$1400 \rightarrow 0$$

Figure 1.2.1 illustrates this example.

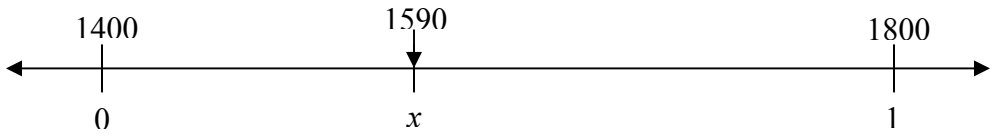


Figure 1.2.1: Determining the common unit values for the SAT scores measure

Using the same method as in the previous section, Enrique and Anna solve a proportion to find x.

$$\frac{|1590-1400|}{|1800-1400|} = \frac{|x-0|}{|1-0|}$$
$$\frac{190}{400} = \frac{x}{1}$$
$$x = 0.475$$

Therefore, an average SAT score of 1590 is converted to a common unit value of 0.475. Enrique and Anna decide to use proportional common units for each of the measures that have a continuous scale.

Converting Categorical Measures
For the categorical measures, Enrique and Anna begin by assigning the best value a 1 and the worst value a 0. Then, Enrique and Anna decide how to apportion the common units. In some cases, apportionment is proportional, while in other cases it is not. They decide to use proportional common units for Nearness to Home, Athletics, and Reputation.

On the other hand, Enrique feels that some measures should not be apportioned proportionately. For example, Enrique and Anna decided that there is a big difference between being ranked nationally and regionally on the *U.S. News & World Report* Ranking measure. Thus, they decide to have the following common units:

Nationally Ranked $\rightarrow$ 1
Regionally Ranked $\rightarrow$ 0.5
Tier 3 $\rightarrow$ 0.25
Tier 4 $\rightarrow$ 0

Enrique prefers a smaller school. Therefore, he assigns the following common units:

Under 3,000 students → 1
3,000-6,000 students → 0.75
6,001-12,000 students → 0.25
Over 12,000 students → 0

Table 1.2.6 contains the results of Enrique and Anna's rescaling of each measure to common units.

Measure	Canisius	Clark	Drexel	Suffolk
Average SAT Score	0.475	0.875	0.750	0.200
U.S. News & World Report Ranking	0.50	1	1	0.25
Room & Board	0.642	0.858	0.311	0.340
Tuition	0.684	0.110	0.453	0.915
Average Daily High Temperature	0.30	0.30	0.70	0.45
Nearness to Home	1	0.5	0	0.5
Athletics	1	0	1	0
Reputation	1	0.5	0.5	1
Size	0.75	1	0	0.75

Table 1.2.6: Each measure rescaled to common units

Q4. From Table 1.2.3, Clark University has the highest average combined SAT score and the highest tuition. Why does it make sense in Table 1.2.6 that Clark has the highest common unit value on one of those measures, but the lowest common unit value on the other?

Q5. Looking at the Nearness to Home measure in Table 1.2.6, what was the most desirable distance to Enrique? What was least desirable?

Q6. Looking at the Athletics measure in Table 1.2.6, what was the most desirable division to Enrique? What was least desirable?

Q7. Looking at the Reputation measure in Table 1.2.6, what was the most desirable reputation to Enrique? What was least desirable?

To review, there are essentially three steps to rescale data to common units.

Step 1: Assign 1 to the best value in the range.
Assign 0 to the worst value in the range.

Step 2: For continuous data, assign intermediate scores proportionally:

$$\text{Scaled Score} = \frac{|\text{score} - \text{least preferred score}|}{|\text{most preferred score} - \text{least preferred score}|}$$

Step 3: For categorical data, assign intermediate scores proportionally or based on your own opinions and values.

1.2.5 Conduct an Interview to Calculate Weights

Next, Enrique and Anna assign weights to each of the measures to reflect the relative importance Enrique attaches to each of them. They decide Anna will interview Enrique. She makes observations to ensure that

Enrique understands the measures he chose and the effects of the weights he assigns to each of them. As a reference tool during the interview, they create Table 1.2.7.

Anna: We have some measures and their ranges for making a decision about your college preference. Focus first on the column of least preferred values. Which one of the measures would you most want to increase from the least preferred value to its most preferred value? For example, is it more important to you to move the SAT score from 1400 to 1800 or to reduce tuition from $35,000 to $25,000?

Enrique: Lower the tuition!

Anna: Are you sure that lowering the tuition to $25,000 is the most important improvement in the whole list?

Enrique: Yes, so I think we should rank tuition number one.

Anna: Enrique, what would be the next most important measure to move from least preferred to most preferred?

Enrique: *U.S. News & World Report* ranking is important, so let's rank that second, and SAT score third.

They continue like this until each measure has been ranked, as shown in Table 1.2.7.

Criteria	Measure	Least preferred	Most preferred	Rank order	Points (0–100)	Weight (Points/Sum)
Academics	Average SAT Score	1400	1800	3		
	U.S. News & World Report Ranking	Tier 4	Nat'l. Rank	2		
Cost	Room & Board	$14,000	$8,000	4		
	Tuition	$35,000	$25,000	1		
Location	Average Daily High Temperature	50° F	70° F	9		
	Nearness to Home	Within 1 hr.	Within 1 day	6		
Social life	Athletics	Div. 3	Div. 1	8		
	Reputation	Party	Balanced	7		
	Size	Over 12,000	Under 3,000	5		
				Sum:		

Table 1.2.7: Ranking and weighting the measures

The next task is to subjectively assign points from 0 to 100 for each measure based on the rank order. The points assigned reflected the relative importance Enrique places on each measure. They continue the interview to assign these points. During this interview process, Anna encourages Enrique to think about the relative importance of moving between the best and worst values of the two measures being considered. This is seen in the next part of the interview.

Anna: Let's start by assigning 100 points to the tuition range, which you've ranked first. Now, you've ranked *U. S. News & World Report* rating second. How important is this rating, from worst to best, compared to reducing the cost of tuition from $35,000 to $25,000? If it's close, you should use a number close to 100.

Enrique: I think it's about 90% as important, so let's use 90 points for that one, and SAT scores are almost as important, so we'll use 85 points for that range.

Table 1.2.8 contains the rest of the points Enrique assigns to each of his measures.

Criteria	Measure	Least preferred	Most preferred	Rank order	Points (0–100)	Weight (Points/Sum)
Academics	Average SAT Score	1400	1800	3	85	
	U.S. News & World Report Ranking	Tier 4	Nat'l. Rank	2	90	
Cost	Room & Board	$14,000	$8,000	4	80	
	Tuition	$35,000	$25,000	1	100	
Location	Average Daily High Temperature	50° F	70° F	9	20	
	Nearness to Home	Within 1 hr.	Within 1 day	6	60	
Social life	Athletics	Div. 3	Div. 1	8	30	
	Reputation	Party	Balanced	7	50	
	Size	> 12,000	< 3,000	5	70	
				Sum:	585	

Table 1.2.8: Enrique's rank and point assignment

The interview continues:

Anna: Enrique, what did you get for the total number of points for all your measures? Once you have the point total, you'll need to divide the points for each measure by this total to get the weight.

Enrique: I got 585 total points. Now I can calculate the weights.

The weights Enrique calculates appear in Table 1.2.9. These were calculated by dividing the points for a particular measure by the total points. For example, Average SAT Score has a point value of 85. So, the weight for this measure is:

$$\frac{85}{585} = 0.145 .$$

Criteria	Measure	Least preferred	Most preferred	Rank order	Points (0–100)	Weight (Points/Sum)
Academics	Average SAT Score	1400	1800	3	85	0.145
	U.S. News & World Report Ranking	Tier 4	Nat'l. Rank	2	90	0.154
Cost	Room & Board	$14,000	$8,000	4	80	0.137
	Tuition	$35,000	$25,000	1	100	0.171
Location	Average Daily High Temperature	50° F	70° F	9	20	0.034
	Nearness to Home	Within 1 hr.	Within 1 day	6	60	0.103
Social life	Athletics	Div. 3	Div. 1	8	30	0.051
	Reputation	Party	Balanced	7	50	0.085
	Size	> 12,000	< 3,000	5	70	0.120
				Sum:	585	1.000

Table 1.2.9: Enrique's assignment of weights to each measure

Next, Anna wants to ensure that Enrique has assigned an appropriate weight to each *criterion*. The interview continues.

Anna: Enrique, what is the total weight for each criterion?

Enrique: I get a total of 0.299 for academics, 0.308 for cost, 0.137 for location, and 0.256 for social life.

Anna: Which criterion has the greatest weight assigned to it?

Enrique: It looks like cost, with 0.308.

Anna: Are there criteria with similar weights?

Enrique: It looks like academics and cost are almost the same.

Anna: Are these the criteria you feel are the most important criteria for choosing a college, and do you think they're about the same in importance?

Enrique: I didn't realize I placed so much importance on academics.

Anna: What did you expect to happen?

Enrique: I thought social life would be at the top of the list!

Anna: Well, you gave athletics only 30 points, reputation 50 points, and size 70 points. Do you want to change anything?

Enrique: No, I really think academics and cost are most important.

1.2.6 Calculate Total Scores

Finally, Enrique and Anna calculate a total score for each school. They use the data from Table 1.2.6, where common units were computed, and the weights calculated in the last column of Table 1.2.9 to calculate a score for each school on each measure. In Table 1.2.10 below, Enrique has calculated the product of the weight and the corresponding common unit:

$$\text{Score} = W \cdot CU$$

For example, the common unit score for the average SAT score at Canisius College is 0.475, and the weight Enrique has assigned to average SAT score is 0.145. Multiplying these two numbers yields 0.069. This value appears opposite SAT score and below Canisius in Table 1.2.10. The rest of the values in Table 1.2.10 are computed in the same way. Then, totaling the scores for each measure for each college yields the total scores that appear in the last row in Table 1.2.10, Now Enrique can see which of his college choices best suits his preferences.

Measure	Weight	Canisius	Clark	Drexel	Suffolk
Average SAT Score	0.145	$0.145 \cdot 0.475 = 0.069$	0.127	0.109	0.029
U.S. News & World Report Ranking	0.154	0.077	0.154	0.154	0.038
Room & Board	0.137	0.088	0.117	0.043	0.046
Tuition	0.171	0.117	0.019	0.077	0.156
Average Daily High Temperature	0.034	0.010	0.010	0.024	0.015
Nearness to Home	0.103	0.103	0.051	0	0.051
Athletics	0.051	0.051	0	0.051	0
Reputation	0.085	0.085	0.043	0.043	0.085
Size	0.120	0.090	0.120	0	0.090
Total Score:	**1.000**	**0.690**	**0.641**	**0.501**	**0.512**

Table 1.2.10: Calculating the total scores of Enrique's schools

1.2.7 Interpreting the Results

Enrique reviews these results carefully. He notices that Drexel and Suffolk have scored much lower than his top-ranked choice, so he excludes them from further study. However, he decides to take a closer look at the relative strengths and weaknesses of Canisius, ranked first, and Clark, ranked second. There is only a 0.049 difference between the two, and he is not sure that it is enough evidence to make this critical life decision.

Q8. What are some reasons why Enrique may not choose Canisius, even though it was ranked first?

Q9. On many of the measures, Clark received better scores than Canisius. Why did Canisius end up having the higher total score?

Q10. Suppose Enrique was offered a scholarship at Clark for $5,000. How do you think this would affect Enrique's decision?

Section 1.3: Judy Purchases a Used Car

Judy is trying to decide which used car to purchase from among four possibilities: a 2006 Honda Civic Hybrid, a 2006 Toyota Prius, and a 2007 Nissan Versa that she has found at dealerships, as well as a 2005 Ford Focus that Judy's uncle Roger is trying to sell by himself. Judy asks her friend Dave to help her structure her thoughts in a consistent manner and to use the steps in the process of multi-criteria decision making (see Section 1.2 for a list of the steps).

With Dave's help, Judy decides that the criteria most important for her choice of a used car are minimizing total cost and maximizing condition, accessories, and aesthetics. They identify two measures for each criterion, as shown in Table 1.3.1.

Criterion	Measures
Total cost	Purchase price Miles per gallon, based on the EPA rating when new
Condition	Odometer reading Body condition
Accessories	Functional air conditioner and heater Sound system
Aesthetics	Color Body design

Table 1.3.1: Judy's list of criteria and measures

Judy and Dave collect data on each of the cars being considered. Their data appear in Table 1.3.2.

Car	Purchase Price	Miles per Gallon	Odometer Reading	Body Condition	Air Conditioner and Heater	Sound System	Color	Body Design
Honda Civic Hybrid	$15,000	43	85,000	Good	Both Work	Radio and CD Players	Red	Sedan
Toyota Prius	$15,500	46	80,000	Good	Both Work	Radio and CD Players	Silver	Sedan
Ford Focus	$7,700	25	95,000	Good	Both Work	Radio, CD, and MP3 Players	Blue	Wagon
Nissan Versa	$11,000	33	65,000	Excellent	Both Work	Radio and CD Players	White	Hatch-back

Table 1.3.2: Judy's data on four used cars

After collecting data and determining the scale range for each measure, Judy and Dave create Table 1.3.3.

Measure	Scale range		Type
Purchase Price	$6,000–$16,000		Continuous
Miles per Gallon	20–50 mpg		Continuous
Odometer Reading	50,000–100,000 miles		Continuous
Body Condition	1	Fair	Categorical
	2	Good	
	3	Excellent	
Functional Air Conditioner and Heater	1	Neither works	Categorical
	2	Only one works	
	3	Both work	
Sound System	1	None	Categorical
	2	Radio only	
	3	Radio and CD player	
	4	Radio, CD, and MP3	
Color	1	Blue	Categorical
	2	Red	
	3	Silver	
	4	White	
	5	Black	
Body Design	1	Wagon	Categorical
	2	Hatchback	
	3	Sedan	

Table 1.3.3: The type and range for each of Judy's measures

Next, Judy and Dave convert each score to a common unit score. They first convert the categorical data into the numerical values given in Table 1.3.4.

Measure	Honda Civic Hybrid	Toyota Prius	Ford Focus	Nissan Versa
Purchase Price	$15,000	$15,500	$7,700	$11,000
Miles per Gallon	43	46	25	33
Odometer Reading	85,000	80,000	95,000	65,000
Body Condition	2	2	2	3
Functional Air Conditioner and Heater	3	3	3	3
Sound System	3	3	4	3
Color	2	3	1	4
Body Design	3	3	1	2

Table 1.3.4: Converted categorical data for Judy's four cars

Then, Judy and Dave convert each continuous measure proportionally (on a scale from 0 to 1) and each categorical measure subjectively, based on Judy's preferences. Table 1.3.5 shows each common unit value. Use this table to answer the questions below.

Measure	Honda Civic Hybrid	Toyota Prius	Ford Focus	Nissan Versa
Purchase price	0.1	0.05	0.83	0.5
Miles per gallon	0.767	0.867	0.167	0.433
Odometer reading	0.3	0.4	0.1	0.7
Body condition	0.5	0.5	0.5	1
A/C and heater	1	1	1	1
Sound system	0.75	0.75	1	0.75
Color	0.5	1	0.75	0
Body design	1	1	0	0.5

Table 1.3.5: Each measure rescaled to common units

Q1. Consider the continuous measures. Determine how Judy and Dave calculated the common unit values for:

a. Purchase Price
b. Miles per Gallon
c. Odometer Reading

Q2. Now consider the categorical measures. Determine Judy's preferences for the following measures based on the information in Table 1.3.5:

a. Body Condition
b. Functional Air Conditioner and Heater
c. Sound System
d. Color
e. Body Design

While the information given in Table 1.3.5 is informative, it does not take Judy's preferences into consideration because each measure is weighted equally.

Therefore, Dave interviews Judy to determine how they would weight each measure. The rank order, points, and weights that came out of this interview can be seen in Table 1.3.6. Use this table to answer the questions below.

Criteria	Measure	Least preferred	Most preferred	Rank order	Points (0–100)	Weight (Points/Sum)
Total cost	Purchase Price	$16,000	$6,000	1	100	0.2
	Miles per Gallon	20 mpg	50 mpg	2	95	0.19
Condition	Odometer Reading	100,000 mi	50,000 mi	5	60	0.12
	Body Condition	1 (fair)	3 (excellent)	3	75	0.15
Accessories	Functional Air Conditioner and Heater	1 (neither works)	3 (both work)	7	35	0.07
	Sound System	1 (none)	4 (radio, CD, MP3)	6	50	0.1
Aesthetics	Color	1 (blue)	5 (black)	8	10	0.02
	Body Design	1 (wagon)	3 (sedan)	3	75	0.15
				Sum =	500	1

Table 1.3.6: Judy's rank ordering, point assignment, and weight calculation for her measures

Q3. Which measure is most important to Judy? How do you know?

Q4. Which measure is least important to Judy? How do you know?

Q5. Which two measures have equal importance to Judy? How do you know?

Q6. How would you describe Judy's feelings towards Purchase Price versus her feelings towards Miles per Gallon?

Q7. How would you describe Judy's feelings towards Miles per Gallon versus her feelings towards Body Condition?

Q8. How were the weights calculated?

Q9. What criterion is most important to Judy? How do you know?

Finally, Judy and Dave calculate the total scores for each car, as shown in Tables 1.3.7 and 1.3.8. Use these tables to answer the questions below.

Measure	Weight	Honda Civic Hybrid	Toyota Prius	Ford Focus	Nissan Versa
Purchase price	0.2	0.1	0.05	0.83	0.5
Miles per gallon	0.19	0.767	0.867	0.167	0.433
Odometer reading	0.12	0.3	0.4	0.1	0.7
Body condition	0.15	0.5	0.5	0.5	1
A/C and heater	0.07	1	1	1	1
Sound system	0.1	0.75	0.75	1	0.75
Color	0.02	0.5	1	0.75	0
Body design	0.15	1	1	0	0.5

Table 1.3.7: Judy's weights and common units for each measure

Measure	Honda Civic Hybrid	Toyota Prius	Ford Focus	Nissan Versa
Purchase price	0.02	0.01	0.166	0.1
Miles per gallon	0.1457	0.1647	0.0317	0.0823
Odometer reading	0.036	0.048	0.012	0.084
Body condition	0.075	0.075	0.075	0.15
A/C and heater	0.07	0.07	0.07	0.07
Sound system	0.075	0.075	0.1	0.075
Color	0.01	0.02	0.015	0
Body design	0.15	0.15	0	0.075
Total Score	**0.58173**	**0.61273**	**0.46973**	**0.63627**

Table 1.3.8: Judy's calculation of the total score for each used car

Q10. How were the scores for each measure calculated?

Q11. Looking at the total scores, which car would you recommend to Judy? Is this choice obvious?

Q12. What significant advantages does the Toyota Prius have over the Nissan Versa?

Q13. What significant advantages does the Nissan Versa have over the Toyota Prius?

When the numeric values are this close, the ultimate answer may be that the decision maker will be equally satisfied with either choice.

Q14. If you were choosing among these cars, which car would you choose? Was your choice impacted by the total scores calculated in this problem?

1.3.1 Using Excel to Calculate Total Scores

An Excel Spreadsheet has an advantage over a calculator. It is able to display the entire matrix of information. Thus, Judy decides to put the data from Table 1.3.7 into an Excel spreadsheet. Her spreadsheet is shown in Figure 1.3.1.

	A	B	C	D	E	F
1			Common Unit Values			
2	**Measure**	**Weight**	**Honda Civic Hybrid**	**Toyota Prius**	**Ford Focus**	**Nissan Versa**
3	**Purchase price**	0.2	0.1	0.05	0.83	0.5
4	**Miles per gallon**	0.19	0.767	0.867	0.167	0.433
5	**Odometer reading**	0.12	0.3	0.4	0.1	0.7
6	**Body condition**	0.15	0.5	0.5	0.5	1
7	**A/C and heater**	0.07	1	1	1	1
8	**Sound system**	0.1	0.75	0.75	1	0.75
9	**Color**	0.02	0.5	1	0.75	0
10	**Body design**	0.15	1	1	0	0.5

Figure 1.3.1: The data from Table 1.3.7 input into a spreadsheet

Column B in the spreadsheet has all of the weights. These are stored in cells B3 through B10. The common unit values for the Honda Civic Hybrid are stored in cells C3 through C10 (Note: to see how to calculate common unit values in Excel, see Appendix A).

Judy wants to use Excel to calculate scores. She knows that there will be a score for each measure, for each car. So she creates another table that looks just like Figure 1.3.1, but she leaves the entries blank, as shown in Figure 1.3.2.

	A	B	C	D	E	F
1			Common Unit Values			
2	Measure	Weight	Honda Civic Hybrid	Toyota Prius	Ford Focus	Nissan Versa
3	Purchase price	0.2	0.1	0.05	0.83	0.5
4	Miles per gallon	0.19	0.767	0.867	0.167	0.433
5	Odometer reading	0.12	0.3	0.4	0.1	0.7
6	Body condition	0.15	0.5	0.5	0.5	1
7	A/C and heater	0.07	1	1	1	1
8	Sound system	0.1	0.75	0.75	1	0.75
9	Color	0.02	0.5	1	0.75	0
10	Body design	0.15	1	1	0	0.5
11						
12			Weight · Common Unit			
13	Measure	Weight	Honda Civic Hybrid	Toyota Prius	Ford Focus	Nissan Versa
14	Purchase price	0.2				
15	Miles per gallon	0.19				
16	Odometer reading	0.12				
17	Body condition	0.15				
18	A/C and heater	0.07				
19	Sound system	0.1				
20	Color	0.02				
21	Body design	0.15				

Figure 1.3.2: Creating an additional table for scores

To calculate the scores, Judy multiplies the common units by the weights. For example, to find the Honda Civic Hybrid's purchase price score, Judy multiplies the weight (0.2) by the common unit value (0.1). In Excel, these values are found in cells B3 and C3, respectively.

To multiplys the value in cell B3 by the value in cell C3, Judy types "=B3*C3" into cell C14. Judy needs to calculate products like this for each value in the table. Fortunately, Excel has an important capability that makes this process much faster.

Once Judy has written the formula, she can use the Fill Handle to copy the same formula into other cells. The Fill Handle is the little blank square at the bottom of a highlighted cell. Judy simply left clicks on the square and drags the formula into a neighboring cell. (Note: Judy could copy and paste the cell into the other cells if she prefers that to the Fill Handle.)

Excel has simple rules for updating the formula. For example, if Judy copies cell C14 to C15, the formula in C15 will be =B4*C4. She has copied the cell to the next row. Excel simply updates the formula from =B3*C3to =B4*C4. In that way, Judy can complete the calculations for C16 through C21, as shown in Figure 1.3.3.

C15 =B4*C4

	A	B	C	D	E	F
1			Common Unit Values			
2	Measure	Weight	Honda Civic Hybrid	Toyota Prius	Ford Focus	Nissan Versa
3	Purchase price	0.2	0.1	0.05	0.83	0.5
4	Miles per gallon	0.19	0.767	0.867	0.167	0.433
5	Odometer reading	0.12	0.3	0.4	0.1	0.7
6	Body condition	0.15	0.5	0.5	0.5	1
7	A/C and heater	0.07	1	1	1	1
8	Sound system	0.1	0.75	0.75	1	0.75
9	Color	0.02	0.5	1	0.75	0
10	Body design	0.15	1	1	0	0.5
11						
12			Weight · Common Unit			
13	Measure	Weight	Honda Civic Hybrid	Toyota Prius	Ford Focus	Nissan Versa
14	Purchase price	0.2	0.02			
15	Miles per gallon	0.19	0.14573			
16	Odometer reading	0.12	0.036			
17	Body condition	0.15	0.075			
18	A/C and heater	0.07	0.07			
19	Sound system	0.1	0.075			
20	Color	0.02	0.01			
21	Body design	0.15	0.15			

Figure 1.3.3: Finding the scores for the Honda Civic Hybrid using Excel formulas

Judy continues in this way to find the scores for the other cars. To save time, she tries to use the Fill Handle to drag the formula from cell C14 to cell D14. However, she notices that the resultant formula is not what she wants. It is "=C3*D3," but she wants "=B3*D3." That is, she wants the value in cell B3 to be the same for each formula in that row. To do so, she uses a $ as an absolute cell reference.

In each calculation, Judy always wants to use the same column B. It has the weights and they apply to all vehicles. By placing a $ sign before the B, Excel knows not to change the column letter as she drags the formula. Now, she instead types the formula "=$B3*C3" in cell C14. Then,
when she drags cell C14 into cell D14, the result is "=$B3*D3". She can then drag the formula in cell C14 into all of the cells of the table Figure 1.3.4 shows the scores for all four cars.

D14 =$B3*D3

	A	B	C	D	E	F
1			Common Unit Values			
2	Measure	Weight	Honda Civic Hybrid	Toyota Prius	Ford Focus	Nissan Versa
3	Purchase price	0.2	0.1	0.05	0.83	0.5
4	Miles per gallon	0.19	0.767	0.867	0.167	0.433
5	Odometer reading	0.12	0.3	0.4	0.1	0.7
6	Body condition	0.15	0.5	0.5	0.5	1
7	A/C and heater	0.07	1	1	1	1
8	Sound system	0.1	0.75	0.75	1	0.75
9	Color	0.02	0.5	1	0.75	0
10	Body design	0.15	1	1	0	0.5
11						
12			Weight · Common Unit			
13	Measure	Weight	Honda Civic Hybrid	Toyota Prius	Ford Focus	Nissan Versa
14	Purchase price	0.2	0.02	0.01	0.166	0.1
15	Miles per gallon	0.19	0.14573	0.16473	0.03173	0.08227
16	Odometer reading	0.12	0.036	0.048	0.012	0.084
17	Body condition	0.15	0.075	0.075	0.075	0.15
18	A/C and heater	0.07	0.07	0.07	0.07	0.07
19	Sound system	0.1	0.075	0.075	0.1	0.075
20	Color	0.02	0.01	0.02	0.015	0
21	Body design	0.15	0.15	0.15	0	0.075

Figure 1.3.4: The scores for the four cars

Q15. Why may Judy decide not to include the A/C and heater measure?

Next, Judy calculates the total score for each car. These values will help her decide which car to purchase. In Excel, the total score for each vehicle is just the sum of the column values. The SUM command in Excel does this calculation. First, Judy calculates the total score for the Honda Civic Hybrid.
In cell C22 she places the following statement:

=SUM(C14:C21).

This tells the spreadhseet to sum the values in all of the cells from C14 through C21, which are the scores for the Honda Civic Hybrid. The colon tells Excel to include all of the cells in between.

Now, to save time, Judy calculates the total score for the Toyota Prius by dragging the value from cell C22 into cell D22 using the Fill Handle.

The results are displayed in Figure 1.3.5. This is exactly what was displayed in Table 1.3.8.

	A	B	C	D	E	F
1			Common Unit Values			
2	**Measure**	**Weight**	**Honda Civic Hybrid**	**Toyota Prius**	**Ford Focus**	**Nissan Versa**
3	**Purchase price**	0.2	0.1	0.05	0.83	0.5
4	**Miles per gallon**	0.19	0.767	0.867	0.167	0.433
5	**Odometer reading**	0.12	0.3	0.4	0.1	0.7
6	**Body condition**	0.15	0.5	0.5	0.5	1
7	**A/C and heater**	0.07	1	1	1	1
8	**Sound system**	0.1	0.75	0.75	1	0.75
9	**Color**	0.02	0.5	1	0.75	0
10	**Body design**	0.15	1	1	0	0.5
11						
12			Weight · Common Unit			
13	**Measure**	**Weight**	**Honda Civic Hybrid**	**Toyota Prius**	**Ford Focus**	**Nissan Versa**
14	**Purchase price**	0.2	0.02	0.01	0.166	0.1
15	**Miles per gallon**	0.19	0.14573	0.16473	0.03173	0.08227
16	**Odometer reading**	0.12	0.036	0.048	0.012	0.084
17	**Body condition**	0.15	0.075	0.075	0.075	0.15
18	**A/C and heater**	0.07	0.07	0.07	0.07	0.07
19	**Sound system**	0.1	0.075	0.075	0.1	0.075
20	**Color**	0.02	0.01	0.02	0.015	0
21	**Body design**	0.15	0.15	0.15	0	0.075
22		**Total Scores:**	**0.58173**	**0.61273**	**0.46973**	**0.63627**

Figure 1.3.5: Judy's spreadsheet calculation of the scores for each used car

Using Excel made the process of finding total scores much faster for Judy. However, there is a method that is even quicker, called SUMPRODUCT.

In Figure 1.3.5, all of the components of the total score are shown. Each cell contains the product Weight · Common Unit. If all Judy wanted was the total score, Excel has a simple command, called SUMPRODUCT. In Figure 1.3.6, Judy placed in cell C11 the command =SUMPRODUCT($B3:$B10,C3:C10).

This command tells Excel to multiply B3 by C3, then B4 by C4, etc., until B10 by C11. Then, it sums these products. This is done in just one command. Judy uses a $ sign so she can copy the contents of C11 into D11 through F11. The form of the command involves stating the range of values in Column B as $B3:$B10. This range is followed by a comma before giving the second range C3:C10. There must be exactly the same number of cells in each range for the command to work.

C11 f_x =SUMPRODUCT($B3:$B10,C3:C10)

	A	B	C	D	E	F
1			Common Unit Values			
2	**Measure**	**Weight**	**Honda Civic Hybrid**	**Toyota Prius**	**Ford Focus**	**Nissan Versa**
3	**Purchase price**	0.2	0.1	0.05	0.83	0.5
4	**Miles per gallon**	0.19	0.767	0.867	0.167	0.433
5	**Odometer reading**	0.12	0.3	0.4	0.1	0.7
6	**Body condition**	0.15	0.5	0.5	0.5	1
7	**A/C and heater**	0.07	1	1	1	1
8	**Sound system**	0.1	0.75	0.75	1	0.75
9	**Color**	0.02	0.5	1	0.75	0
10	**Body design**	0.15	1	1	0	0.5
11	**Total Scores:**		**0.58173**	**0.61273**	**0.46973**	**0.63627**

Figure 1.3.6: Using the command SUMPRODUCT to calculate total scores

Chapter 1 (MCDM) Homework Questions

1. Olivia wants to pursue a career in medicine, but she is not sure which profession would be best for her. After some preliminary research, she narrows her choices to physician, nurse, and pharmacist. Olivia decides to consider four criteria to help structure her decision: professional preparation, personal fulfillment, financial compensation, and lifestyle. The table below shows these criteria and the measures she has decided to use for each.

Criterion	Measure	Type of Scale	Type of Data
Professional Preparation	Schooling		
	Internship		
	Difficulty		
Personal Fulfillment	Job satisfaction		
	Personal interest		
Financial Compensation	Initial salary		
	Median salary		
Lifestyle	Likely schedule		
	Maternity leave		
	Prestige		

   a. Decide which type of scale would be appropriate for each measure, either *continuous-natural* or *categorical-constructed*.

   b. Determine which of the data will have to be collected through *research* and what will be based on personal *opinion*.

   c. The table below shows some of the data Olivia has collected for the professional preparation criterion. Based on the scale ranges, determine what you would consider most preferred and least preferred for each measure.

Criterion	Measure	Scale Range	Physician (M.D.)	Nurse (R.N.)	Pharmacist (Pharm.D.)
Professional Preparation	Schooling (years)	2-8	8	4	6
	Internship (years)	0-4	3	0	1
	Difficulty (rank)	1-3	1	3	2

   d. What else must be done before obtaining common unit values?

   e. Fill in the following table with scores scaled to common units.

Criterion	Measure	Physician (M.D.)	Nurse (R.N.)	Pharmacist (Pharm.D.)
Professional Preparation	Schooling			
	Internship			
	Difficulty			

f. Suppose Olivia weights Schooling at 0.109, Internship at 0.091, and Difficulty at 0.073. Complete the following table with the weighted scores.

Criterion	Measure	Physician (M.D.)	Nurse (R.N.)	Pharmacist (Pharm.D.)
Professional Preparation	Schooling			
	Internship			
	Difficulty			

2. Rana is trying to decide what type of engineering to study in college. She obtained the following data after an initial search, but now she wants to take a more organized and methodical approach.

	Mechanical	Electrical	Chemical	Industrial
Starting salary ($)	54,128	55,292	59,361	55,067
Median salary ($)	75,130	82,090	84,240	73,490
Job Satisfaction (%)	83.0	79.9	83.8	80.5

a. Create a list of four criteria she could use to help make this decision. Try to find criteria that would use these data.

b. Identify two measures for each criterion.

3. Give an example of a measure that uses a continuous scale, but might not be converted to common units proportionally. Explain your answer.

4. Give an example of a measure that uses a categorical scale, but might not be converted to common units proportionally. Explain your answer.

5. In problem 1.1 of the chapter, Isabelle Nueva is helping her mother and father decide on the best cell phone plan for her family.
   a. What additional measures do you think should be considered?

   b. Add and describe a categorical measure for the problem and create 3 categories for this new measure.

   c. Add and describe a numerical measure for the problem.

6. In problem 1.2 of the chapter, Enrique Ramirez is selecting a college to attend.
   a. What additional measures do you think should be considered?

   b. Add and describe a categorical measure for the problem and create 3 categories for this new measure.

   c. Add and describe a numerical measure for the problem.

7. In problem 1.3 of the chapter, Judy is choosing which used car to purchase from among four possibilities.
   a. What additional measures do you think should be considered?

   b. Add and describe a categorical measure for the problem and create 3 categories for this new measure.

   c. Add and describe a numerical measure for the problem.

8. A high school student wants to buy a digital camera. Checking the experts' recommendations, she creates a list of important features and ranks them as follows. She ranks Price as the most important measure and, therefore, assigns 100 points it. Brand name is slightly less important than price. It is ranked 2nd and she assigns 90 points to it. She thinks that having an Anti-shake system is much less important than brand name and assigns 60 points to it. Size of view screen is ranked below anti-shake system and has a little bit less importance, thus she assigned it 55 points. Finally, ease of use is the least important factor with 40 points. Calculate the weight assigned to each measure.

Measure	Rank	Points	Weight
Size of view screen	4	55	
Price	1	100	
Brand name	2	90	
Anti-shake system	3	60	
Easy to use	5	40	
Total			

9. Suppose you are looking to buy a digital camera for yourself.
   a. Suggest and add a relevant categorical measure in the table below. Describe the new measure.

   b. Suggest and add a relevant numerical measure in the table below. Describe the new measure.

   c. Use your personal preferences and rank the measures. Then, assign points to each measure and calculate the weight of each measure.

	Assign		Calculate
Measure	**Rank**	**Points**	**Weight**
Size of view screen			
Price			
Brand name			
Anti-shake system			
Easy to use			
New categorical measure:			
New numerical measure:			
Total			

10. Kim is interested in purchasing a desktop computer for her office. After reviewing the specification of different models, she ended up with the following measures. Classify each measure as numerical or categorical.

Measure	Type: Numeric or Categorical
Computational power	
Monitor size	
Years of warranty	
Operating system	
Price	

11. A high school has selected one of its students to be the chair of a committee planning a class trip. One of her first responsibilities is to pick a co-chair for planning the trip. Suggest two measures for each criterion. Specify the type of each measure.

Criterion	Measure	Type: Numerical or Categorical
Knowledge		
Reliability		
Personality		

12. Jay is a movie fan and is considering two companies that offer DVD rental membership, Netco and DVDco. Netco is a completely web-based company but DVDco has branch stores in addition to their web site.

    a. Having studied the web sites of the two main companies, Jay summarized the measures and data as follows:

Measure	Netco	DVDco
Number of available movies	100,000	75,000
Price	10	8
Web streaming	yes	no
Number of DVDs that can be out at the same time	1	2
Availability of a recommendation system	no	yes
In-store return	no	yes

Specify the range for each measure and then determine the common unit for each of them. Insert the common units in the following table.

Measure	Netco	DVDco
Number of available movies		
Price		
Web streaming		
Number of DVDs that can be out at the same time		
Availability of a recommendation system		
In-store return		

After considering the measures, Jay ranked the measures and assigned the points as indicated below. Calculate the weight of each measure based on the points.

Measure	Rank	Points	Weight
Number of available movies	4	75	
Price	1	100	
Web streaming	2	90	
Number of DVDs that can be out at the same time	5	60	
Availability of a recommendation system	6	50	
In-store return	3	85	

b. For each alternative, calculate the product of the weight and the corresponding common unit for each measure. Determine the total score for each alternative.

Measure	Netco	DVDco
Number of available movies		
Price		
Web streaming		
Number of DVDs that can be out at the same time		
Availability of a recommendation system		
In-store return		
Total Score		

c. Which alternative is ranked 1st, and what measures contribute the most to it being ranked 1st?

13. Sam and his wife were just married and are looking for an apartment in a safe area close to Sam's school. After discussing their preferences, they came up with the following measures that are very important to them.

Measure	Description
Spaciousness	Size and design
Price	Monthly rental
Condition	Freshly painted, floors, age of appliances
Apartment building rating	Based on rating of previous tenants' rating in www.apartmentrating.com

a. After searching in a 10-mile radius around his school, they ended up with the following three apartments they like. Sam summarized the data as follows:

Measure	Ap1	Ap2	Ap3
Spaciousness	Good	Medium	Poor
Price ($/month)	700	650	550
Condition (0-1)	0.6	0.9	0.7
Apartment building rating (between 1 and 5)	4	4.5	3.8

b. Specify the range for each measure and then determine the common unit for each of them. Insert the common units in the following table.

Measure	Ap1	Ap2	Ap3
Spaciousness			
Price			
Condition(0-1)			
Apartment building rating			

c. After considering the measures, Sam and his wife ranked the measures as in the following table. Use the assigned points to calculate the weights.

Measure	Rank	Points	Weight
Spaciousness	3	70	
Price	1	100	
Condition(0-1)	2	90	
Apartment Building Rating	4	50	

d. For each alternative, calculate the product of the weight and the corresponding common unit for each measure. Determine the total score for each alternative.

Measure	Ap1	Ap2	Ap3
Spaciousness			
Price			
Condition			
Apartment Building Rating			
Total Score			

e. Which alternative is ranked 1st and what measures contribute the most to it being ranked 1st?

14. James and George are seeking a team member for their capstone project. It is a very demanding project that requires a wide range of skills. To help evaluate potential teammates, they created the following list of measures.

Measure
Writing Skills
GPA of Math courses
Total GPA
Reliability and commitment
Communication skills

a. After considering all their classmates who were not yet assigned to any project, they ended up with following three people. They summarized the data for these three as follows:

Measure	Ed	Ken	Thad
Writing skills	Excellent	Acceptable	Good
GPA in math courses	3.8	3.9	3.5
Total GPA	3.6	3.8	3.7
Reliability and commitment	Acceptable	Good	Good
Communication skills	Good	Excellent	Excellent

b. Specify the range for each measure and then determine the common unit for each of them. Insert the common units in the following table.

Measure	Ed	Ken	Thad
Writing skills			
GPA in math courses			
Total GPA			
Reliability and commitment			
Communication skills			

c. They are not sure how to rank the measures. Based on your personal preferences, rank the measures and fill out the rest of table.

Measure	Rank	Points	Weight
Writing skills			
GPA in math courses			
Total GPA			
Reliability and commitment			
Communication skills			
	Total		

d. For each alternative, calculate the product of the weight and the corresponding common unit for each measure. Determine total score for each alternative.

Measure	Ed	Ken	Thad
Writing skills			
GPA in math courses			
Total GPA			
Reliability and commitment			
Communication skills			
Total Score			

e. Which alternative is ranked 1st and what measures contribute the most to him being ranked 1st?

15. Neil is trying to find a location in Michigan to open a convenience store. Location is very important for convenience stores. Thus, he wants to be very precise in this process. After talking to some consultants and other store managers, he plans to use the following measures.

Measure	Description
Traffic through intersection	Daily number of the cars passing the intersection
Population within 2 mile	Total population over the age of 15
Distance to the nearest competitor	Miles to nearest convenience store
Cost of the property	Purchase price of property

a. After considering all available properties in the area, he ends up with the following three locations. The data for these three locations is summarized below.

Measure	L1	L2	L3
Traffic through intersection (vehicles)	16,000	15,000	19,000
Population within 2 miles	50,000	45,000	55,000
Distance to the nearest competitor (miles)	1.5	2	0.5
Cost of the property ($)	210,000	180,000	250,000

b. Specify the range for each measure and then determine the common unit for each of them. Insert the common units in the following table.

Measure	L1	L2	L3
Traffic through intersection			
Population within 2 mile			
Distance to the nearest competitor			
Cost of the property			

c. After considering the measures, he ranks the measures as in the following table. Use assigned points to calculate the weights.

Measure	Rank	Point	Weight
Traffic through intersection	2	85	
Population within 2 mile	3	80	
Distance to the nearest competitor	4	70	
Cost of the property	1	100	
Total			

d. For each alternative, calculate the product of the weight and the corresponding common unit for each measure. Determine total score for each alternative.

Measure	L1	L2	L3
Traffic through intersection			
Population within 2 mile			
Distance to the nearest competitor			
Cost of the property			
Total Score			

e. Which alternative is ranked 1st and what measures contribute the most to it being ranked 1st?

Chapter 1 Summary

What have we learned?

We have learned that the multi-criteria decision making process provides a framework for making a subjective decision when considering several alternatives, each of which has advantages and disadvantages. As the person making the decision, you must structure the decision. What criteria or objectives will be considered? What measures of your criteria will be included? How will you rank and weight these measures to help make a decision that is best for your values and priorities?

This process allows for direct comparison and evaluation of complex alternatives. The steps are as follows:

1. Identify Criteria and Measures
2. Collect Data
3. Find the Range of Each Measure
4. Rescale Each Measure to a Common Unit
5. Conduct an Interview to Calculate Weights
6. Calculate Total Scores
7. Interpret Results

Terms

Categorical Measure	A measure whose scores are classifications
Common Unit	A value that varies from 0 to 1, where 0 always represents the worst value, 1 the best value, and intermediate values are found using a proportional scale
Continuous Measure	A measure whose scores are numeric values that can take on any value in a certain range
Criteria	Objectives or aspects of the alternatives that you wish to either maximize or minimize
Measure	A trait that will quantify an aspect of a criterion
Proportional Scale	The rescaled score for intermediate values of continuous measures (calculated by dividing the difference between the particular score and the least preferred score by the scale range)
Scale Range	The range of possible values for each measure.
Total score	For each alternative, multiply the rescaled score by the weight for each measure. The sum of all these weighted, rescaled scores is the total score.
Weighted Scores	The rescaled score for each measure, weighted according to its importance (calculated by multiplying each scaled score by the corresponding weight of the measure)

Chapter 1 (MCDM) Objectives

You should be able to:

- List the sequence of steps in the multi-criteria decision making process
- Explain the purpose of each step in the process
- Identify criteria you will use to choose between several alternatives
- Select measure(s) for each criterion
- Distinguish between categorical and continuous measures
- Determine scale types and ranges for measures
- Scale scores
- Rescale scores to common units
- Weight scores for each measure
- Calculate a total score for each alternative
- Evaluate the results of the multi-criteria decision making process by comparing the strengths and weaknesses of the top two alternatives

Chapter 1 Study Guide

1. Explain why the Multi-Criteria Decision Making (MCDM) process is useful.

2. Discuss the differences between a *criterion* and a *measure*.

3. When choosing between the same alternatives, why might you and a classmate, both using MCDM, come to a different decision?

4. Compare and contrast *continuous* and *categorical* measures.

5. Give an example of a scale range in which one end is most preferable for you, but the other end may be preferable to a classmate. Explain.

6. Why do we scale all scores between zero and one?

7. Describe how scores are scaled differently for continuous and categorical measures.

8. Describe how scaled scores are rescaled to common units differently for continuous and categorical measures.

9. Identify which steps in MCDM involve you inserting your own preferences and priorities into the process and describe how this occurs?

10. What role do the weights of the measures play in determining which alternative is the best?

11. Describe the process that occurs from collecting raw data for measures to obtaining a total score for an alternative.

12. Should you always choose the alternative with the highest total score?

References

Lenhart, A. (2010). *Teens, cell phones and texting*. Pew Internet & American Life Project: Pew Research Center.

Appendix A: Alternative Section 1.2 (Enrique Ramirez Chooses a College) with Excel

Enrique Ramirez has been accepted at four colleges: Canisius College in Buffalo, NY; Clark University in Worcester, MA; Drexel University in Philadelphia, PA; and Suffolk University in Boston, MA. Now he must decide which one to attend.

Enrique asks his friend Anna for help. Enrique and Anna realize that there are many different issues to consider when making this decision. They also realize that the issues of interest to Enrique and their relative importance are not the same as those for Anna.

To make this decision, Enrique, with the help of Anna, follows the steps of MCDM that were presented in the previous section. These steps are given in Table 1.2.1.

General Steps	Descriptions for this Particular Decision
1. Identify Criteria and Measures	First, generate a list containing general criteria that are important when choosing a college. These criteria will be broad in nature and will be based on objective and subjective goals. Next, specify at least one measure for each criterion.
2. Collect Data	For each college, collect the data for each measure.
3. Find the Range of Each Measure	Specify a reasonable scale for each measure.
4. Rescale Each Measure to a Common Unit	Rescale each measure to common units from 0 to 1, with 0 being the worst alternative and 1 being the best alternative.
5. Conduct an Interview to Calculate Weights	With the help of an interviewer, rank-order the measures, assign points from 0 to 100 to each measure, and calculate a proportional weight between 0 and 1 for each measure.
6. Calculate Total Scores	Calculate a total weighted score for each college. These weights will yield a ranking of the colleges, allowing you to identify the best option based on your preferences.
7. Interpret Results	Review the results to understand the strengths and weaknesses of your top alternatives before finalizing your decision.

Table 1.2.1: Steps of MCDM

1.2.1 Identify Criteria and Measures

With Anna's help, Enrique decides that academics, cost, location, and social life are the criteria most critical in his choice of a school. Next, Enrique and Anna take his list of four criteria and specify two or three measures for each criterion. His criteria and measures are given in Table 1.2.2.

Criteria	Measures
Academics	Average SAT Score (based on last year's freshman class)
	U.S. News & World Report Ranking
Cost	Room & Board (annual)
	Tuition (annual)
Location	Average Daily High Temperature

	Nearness to Home
Social Life	Athletics
	Reputation
	Size

Table 1.2.2: Enrique's criteria and measures

1.2.2 Collect Data

For each measure, Enrique and Anna collect data, which is listed in Table 1.2.3. Some of the measures are naturally categorical. For example, *U.S. News & World Report* ranks schools into four categories:

1. Nationally ranked
2. Regionally ranked
3. Regionally tier 3
4. Regionally tier 4

Enrique and Anna divide Athletics into three categories:

1. Division 1
2. Division 2
3. Division 3

Similarly, they divide Reputation into three categories:

1. Seriously academic
2. Balanced academics and social life
3. Party school

The data for the remaining measures are numerical values. Enrique and Anna are able to find the average values for SAT score and daily high temperature and the exact values for room and board cost, tuition cost, nearness to home, and size.

Measure	Canisius	Clark	Drexel	Suffolk
Average SAT Score	1590	1750	1700	1480
U.S. News & World Report Ranking	22nd (regional)	91st (national)	109th (national)	Tier 3 (regional)
Room & Board	$10,150	$8,850	$12,135	$11,960
Tuition	$28,157	$33,900	$30,470	$25,850
Average Daily High Temperature	56°	56°	64°	59°
Nearness to Home	297 mi	157 mi	81 mi	191 mi
Athletics	Division 1	Division 3	Division 1	Division 3
Reputation	Balanced	Seriously academic	Seriously academic	Balanced
Size	3,300 students	2,175 students	12,348 students	4,985 students

Table 1.2.3: Raw data for Enrique's four schools

1.2.3 Find the Range of Each Measure

Next, Enrique and Anna choose an appropriate scale for each of the nine measures. Some of the measures are continuous (e.g., SAT score), while others are categorical (e.g., athletics).

For the Nearness to Home measure, Enrique believes that exact mileage is not important, but rather broad ranges of mileage better represent his concerns. Therefore, Enrique and Anna convert this measure from continuous to categorical.

Q1. Looking at Table 1.2.4, what other measure was converted from continuous to categorical?

Enrique and Anna also realize that the range of each scale is important. For example, the theoretical range of the average combined SAT score is 600–2400, but in actuality, the range of the average combined SAT score at the colleges Enrique is considering is 1480–1750, which is a much narrower range. Enrique and Anna decide that it is much more realistic to use a range that is close to the actual range.

Q2. In the previous section, the ranges for the continuous measures were simply the ranges of the data collected. In this section, the ranges are expanded slightly. For example, instead of the SAT range staying as 1480-1750, Enrique and Anna choose the range 1400-1800. Why might one prefer to use the ranges of the data collected? Why might one prefer to round the ranges?

Q3. Looking at Table 1.2.4, for what other measures do Enrique and Anna create realistic ranges? Do you agree with their ranges? Why or why not?

The scale range and type of each measure are given in Table 1.2.4.

Measure	Scale range	Type
Average SAT Score	1400–1800	Continuous
U.S. News & World Report Ranking	Nationally Ranked	Categorical
	Regionally Ranked	
	Regionally Tier 3	
	Regionally Tier 4	
Room & Board	$8,000–$14,000	Continuous
Tuition	$25,000–$35,000	Continuous
Average Daily High Temperature	50˚–70˚F	Continuous
Nearness to Home	Within 1 hr. Drive (50-100 mi)	Categorical
	Within 4 hr. Drive (101–200 mi)	
	Within a Day's Drive (201–300 mi)	
Athletics	Division 1	Categorical
	Division 2	
	Division 3	
Reputation	Seriously Academic	Categorical
	Balanced Academics and Social Life	
	Party School	
Size	Under 3,000 students	Categorical
	3,001–6,000 students	
	6,001–12,000 students	
	Over 12,000 students	

Table 1.2.4: Types and ranges of measures

Before continuing, Enrique and Anna convert the values of the categorical measures into the numerical values based on the ranges of each measure. The converted data for the categorical measures are given in Table 1.2.5.

Measure	Canisius	Clark	Drexel	Suffolk
Average SAT Score	1590	1750	1700	1480
U.S. News & World Report Ranking	2	1	1	3
Room & Board	$10,150	$8,850	$12,135	$11,960
Tuition	$28,157	$33,900	$30,470	$25,850
Average Daily High Temperature	56˚	56˚	64˚	59˚
Nearness to Home	3	2	1	2
Athletics	1	3	1	3
Reputation	2	1	1	2
Size	2	1	4	2

Table 1.2.5: Converted categorical data for Enrique's four schools

1.2.4 Rescale Each Measure to a Common Unit

Once Enrique and Anna choose appropriate scales for each of the measures, Anna reminds Enrique that if they compared the data in its current form, it would be like comparing apples to oranges. They decide to convert the data to common units. To do so, they assign 1 to the best value and 0 to the worst value in the range of each measure. Recall from the previous section, the method for determining intermediate values differs for continuous and categorical measures.

Converting Continuous Measures
For the continuous measures, Enrique and Anna use a proportional scale. For example, the Average SAT Score at Canisius is 1590. The range for this measure is 1400–1800, so 1400 (the least desirable score) should be converted to 0 and 1800 (the most desirable score) to 1. But what should the proportional value for Canisius be?

$$1800 \to 1$$
$$1590 \to x$$
$$1400 \to 0$$

Figure 1.2.1 illustrates this example.

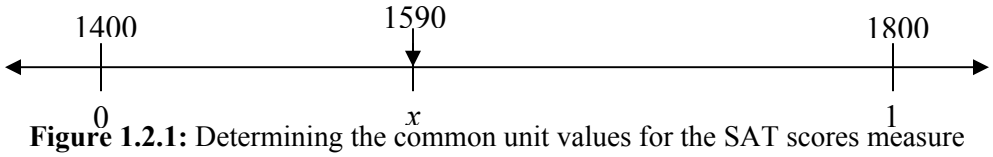


Figure 1.2.1: Determining the common unit values for the SAT scores measure

Using the same method as in the previous section, Enrique and Anna solve a proportion to find x.

$$\frac{|1590-1400|}{|1800-1400|} = \frac{|x-0|}{|1-0|}$$
$$\frac{190}{400} = \frac{x}{1}$$
$$x = 0.475$$

Therefore, an average SAT score of 1590 is converted to a common unit value of 0.475. Enrique and Anna decide to use proportional common units for each of the measures that have a continuous scale.

Enrique and Anna know a similar process could be used to convert the remaining values to common unit values. However, they realize it is more efficient to use a spreadsheet application, such as Microsoft Excel to find these values. Figure 1.2.2 shows the data of Table 1.2.5 in a spreadsheet format. Note: there are two empty rows between each measure, which will be utilized later, and the lines containing data have been shaded to make the worksheet easier to read.

	A	B	C	D	E
1	**Measure**	**Canisius**	**Clark**	**Drexel**	**Suffolk**
2	**Average SAT Score**	**1590**	**1750**	**1700**	**1480**
3					
4					
5	***U.S. News & World Report* Ranking**	**2**	**1**	**1**	**3**
6					
7					
8	**Room & Board**	**10150**	**8850**	**12135**	**11960**
9					
10					
11	**Tuition**	**28157**	**33900**	**30470**	**25850**
12					
13					
14	**Average Daily High Temperature**	**56**	**56**	**64**	**59**
15					
16					
17	**Nearness to Home**	**3**	**2**	**1**	**2**
18					
19					
20	**Athletics**	**1**	**3**	**1**	**3**
21					
22					
23	**Reputation**	**2**	**1**	**1**	**2**
24					
25					
26	**Size**	**2**	**1**	**4**	**2**
27					
28					

Figure 1.2.2: Data in a spreadsheet format

To find the common unit values, Enrique and Anna use the least preferred and most preferred values. Therefore, they add these values to the worksheet. In Figure 1.2.3, these values are added for only the continuous measures (the categorical measures will be discussed later).

	A	B	C	D	E	F	G	H
1	Measure	Canisius	Clark	Drexel	Suffolk		Least Preferred Value	Most Preferred Value
2	Average SAT Score	1590	1750	1700	1480		1400	1800
3								
4								
5	*U.S. News & World Report* Ranking	2	1	1	3			
6								
7								
8	Room & Board	10150	8850	12135	11960		14000	8000
9								
10								
11	Tuition	28157	33900	30470	25850		35000	25000
12								
13								
14	Average Daily High Temperature	56	56	64	59		50	70
15								
16								
17	Nearness to Home	3	2	1	2			
18								
19								
20	Athletics	1	3	1	3			
21								
22								
23	Reputation	2	1	1	2			
24								
25								
26	Size	2	1	4	2			
27								
28								

Figure 1.2.3: Data with least and most preferred values for continuous measures

Recall, to find the common unit value for the Average SAT Score at Canisius, Enrique and Anna performed the following calculation:

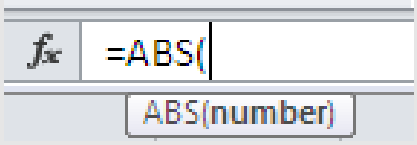
$$x = \frac{|1590 - 1400|}{|1800 - 1400|}$$

$$x = \frac{190}{400}$$

$$x = 0.475$$

To do this calculation in Excel, Enrique and Anna need to use the absolute value function. To do so, they click on cell B3 (the cell below 1590) and typ in "=ABS(". This function takes the absolute value of the number in the parentheses.

When writing any function in Excel, begin with an equal sign (=).



The value of $\frac{|1590 - 1400|}{|1800 - 1400|}$ could be found in Excel using the following:

=ABS(1590-1400)/ABS(1800-1400).

However, Enrique and Anna remember that Excel can reference other values in the spreadsheet. For example, rather than type in 1590, Enrique can type in or click on cell B2 (the name of the cell where 1590 is located). Therefore, Enrique and Anna could find the common unit value for the Average SAT Score at Canisius using the following:

=ABS(B2-G2)/ABS(H2-G2)

Q4. Why may it be beneficial to reference cell values rather than type in the numerical values?

Figure 1.2.4 shows this formula and its value in Excel.

B3 f_x =ABS(B2-G2)/ABS(H2-G2)

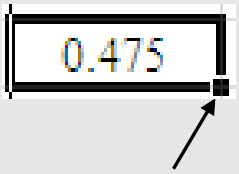
	A	B	C	D	E	F	G	H
1	Measure	Canisius	Clark	Drexel	Suffolk		Least Preferred Value	Most Preferred Value
2	Average SAT Score	1590	1750	1700	1480		1400	1800
3	Common Unit Values	0.475						
4								
5	*U.S. News & World Report* Ranking	2	1	1	3			
6								
7								
8	Room & Board	10150	8850	12135	11960		14000	8000
9								
10								
11	Tuition	28157	33900	30470	25850		35000	25000
12								
13								
14	Average Daily High Temperature	56	56	64	59		50	70
15								
16								
17	Nearness to Home	3	2	1	2			
18								
19								
20	Athletics	1	3	1	3			
21								
22								
23	Reputation	2	1	1	2			
24								
25								
26	Size	2	1	4	2			
27								
28								

Figure 1.2.4: Finding the common unit value for the Average SAT Score at Canisius in Excel

Q5. Using Excel, convert the remaining SAT score values to common unit values. Check your answer by hand.

Enrique and Anna noticed these formulas are repetitive, referencing the same values over and over again (e.g., the least preferred and most preferred values). To make typing in the formulas more efficient, Enrique and Anna use the Excel feature called the *Fill Handle*.

The Fill Handle



- The Fill Handle is the small black square in the bottom right of the active cell.
- It is used to drag a formula into a neighboring cell.
- To drag a formula, left click on the Fill Handle and drag to a neighboring cell (up, down, left, or right).
- To see the formulas written out for each of the cells, hold down the Ctrl key and hit ~; complete the same process to undo this.

Q6. Use the Fill Handle to drag the formula from cell B3 into cells C3, D3, and E3. What do you notice?

C3 =ABS(C2-H2)/ABS(I2-H2)

	A	B	C	D	E	F	G	H
1	Measure	Canisius	Clark	Drexel	Suffolk		Least Preferred Value	Most Preferred Value
2	Average SAT Score	1590	1750	1700	1480		1400	1800
3	Common Unit Values	0.475	0.02778	#DIV/0!	#DIV/0!			
4								
5	*U.S. News & World Report* Ranking	2	1	1	3			
6								
7								
8	Room & Board	10150	8850	12135	11960		14000	8000
9								
10								
11	Tuition	28157	33900	30470	25850		35000	25000
12								
13								
14	Average Daily High Temperature	56	56	64	59		50	70
15								
16								
17	Nearness to Home	3	2	1	2			
18								
19								
20	Athletics	1	3	1	3			
21								
22								
23	Reputation	2	1	1	2			
24								
25								
26	Size	2	1	4	2			
27								
28								

Figure 1.2.5: Dragging the formula into neighboring cells

Enrique and Anna find that the formulas in the new cells are incorrect, as seen in Figure 1.2.5. Looking carefully at the formula in cell C3, they notice that Excel automatically moved each cell reference over as the Fill Handle was dragged over. To fix these errors, they use another feature called *absolute cell reference*.

Absolute Cell Reference

- The default in Excel is relative reference, so if you drag a formula referring to cell A1 to, say cell E20 from cell E19, it would then refer to cell A2.
- Dollar signs are used for absolute cell references. The first dollar sign freezes the column; the second dollar sign freezes the row.
- For example, the cell A1 is an absolute reference to the data, formula, or string in cell A1 of a worksheet. This means that it will still refer to cell A1 if you drag out a formula that refers to it, or otherwise copy a formula to another cell.
- For a short cut, rather than type in the dollar signs, simply click on the cell to be referenced and press F4.

=ABS(B2-G2)/ABS(H2-G2)

In this example, the cells that need to be referenced each time are the least preferred and most preferred values (cells G2 and H2, respectively). The only cell reference that should change when the formula is dragged is the cell with the Average SAT Score. Thus, this reference (B2) should not have dollar signs. See Figure 1.2.6 for the new formula dragged into its neighboring cells.

B3 =ABS(B2-G2)/ABS(H2-G2)

	A	B	C	D	E	F	G	H
1	Measure	Canisius	Clark	Drexel	Suffolk		Least Preferred Value	Most Preferred Value
2	Average SAT Score	1590	1750	1700	1480		1400	1800
3	Common Unit Values	0.475	0.875	0.75	0.2			
4								
5	*U.S. News & World Report* Ranking	2	1	1	3			
6								
7								
8	Room & Board	10150	8850	12135	11960		14000	8000
9								
10								
11	Tuition	28157	33900	30470	25850		35000	25000
12								
13								
14	Average Daily High Temperature	56	56	64	59		50	70
15								
16								
17	Nearness to Home	3	2	1	2			
18								
19								
20	Athletics	1	3	1	3			
21								
22								
23	Reputation	2	1	1	2			
24								
25								
26	Size	2	1	4	2			
27								
28								

Figure 1.2.6: Dragging the formula with absolute cell references into neighboring cells

Q7. Using Excel, convert each of the remaining continuous measure values to common unit values.

Converting Categorical Measures

For the categorical measures, Enrique and Anna begin by assigning the best value a 1 and the worst value a 0. Then, Enrique and Anna decide how to apportion the common units. In some cases, apportionment is proportional, while in other cases it is not. They decide to use proportional common units for Nearness to Home, Athletics, and Reputation.

On the other hand, Enrique feels that some measures should not be apportioned proportionately. For example, Enrique and Anna decided that there is a big difference between being ranked nationally and regionally on the *U.S. News & World Report* Ranking measure. Thus, they decide to have the following common units:

Nationally Ranked $\rightarrow$ 1
Regionally Ranked $\rightarrow$ 0.5
Tier 3 $\rightarrow$ 0.25
Tier 4 $\rightarrow$ 0

Enrique prefers a smaller school. Therefore, he assigns the following common units:

Under 3,000 students $\rightarrow$ 1
3,000-6,000 students $\rightarrow$ 0.75
6,001-12,000 students $\rightarrow$ 0.25
Over 12,000 students $\rightarrow$ 0

Table 1.2.6 contains the results of Enrique and Anna's rescaling of each measure to common units.

Measure	Canisius	Clark	Drexel	Suffolk
Average SAT Score	0.475	0.875	0.750	0.200
U.S. News & World Report Ranking	0.50	1	1	0.25
Room & Board	0.642	0.858	0.311	0.340
Tuition	0.684	0.110	0.453	0.915
Average Daily High Temperature	0.30	0.30	0.70	0.45
Nearness to Home	1	0.5	0	0.5
Athletics	1	0	1	0
Reputation	1	0.5	0.5	1
Size	0.75	1	0	0.75

Table 1.2.6: Each measure rescaled to common units

Q8. From Table 1.2.3, Clark University has the highest average combined SAT score and the highest tuition. Why does it make sense in Table 1.2.6 that Clark has the highest common unit value on one of those measures, but the lowest common unit value on the other?

Q9. Looking at the Nearness to Home measure in Table 1.2.6, what was the most desirable distance to Enrique? What was least desirable?

Q10. Looking at the Athletics measure in Table 1.2.6, what was the most desirable division to Enrique? What was least desirable?

Q11. Looking at the Reputation measure in Table 1.2.6, what was the most desirable reputation to Enrique? What was least desirable?

Enrique and Anna do not need to do any calculations to find the common units for the categorical measures. Therefore, these values can simply be typed into Excel. Figure 1.2.8 shows the completed Excel worksheet.

	A	B	C	D	E	F	G	H
1	**Measure**	**Canisius**	**Clark**	**Drexel**	**Suffolk**		**Least Preferred Value**	**Most Preferred Value**
2	**Average SAT Score**	**1590**	**1750**	**1700**	**1480**		1400	1800
3	Common Unit Values	0.475	0.875	0.75	0.2			
4								
5	***U.S. News & World Report* Ranking**	**2**	**1**	**1**	**3**			
6	Common Unit Values	0.5	1	1	0.25			
7								
8	**Room & Board**	**10150**	**8850**	**12135**	**11960**		14000	8000
9	Common Unit Values	0.6416667	0.85833	0.31083	0.34			
10								
11	**Tuition**	**28157**	**33900**	**30470**	**25850**		35000	25000
12	Common Unit Values	0.6843	0.11	0.453	0.915			
13								
14	**Average Daily High Temperature**	**56**	**56**	**64**	**59**		50	70
15	Common Unit Values	0.3	0.3	0.7	0.45			
16								
17	**Nearness to Home**	**3**	**2**	**1**	**2**			
18	Common Unit Values	1	0.5	0	0.5			
19								
20	**Athletics**	**1**	**3**	**1**	**3**			
21	Common Unit Values	1	0	1	0			
22								
23	**Reputation**	**2**	**1**	**1**	**2**			
24	Common Unit Values	1	0.5	0.5	1			
25								
26	**Size**	**2**	**1**	**4**	**2**			
27	Common Unit Values	0.75	1	0	0.75			
28								

Figure 1.2.8: Completed Excel worksheet with common unit values

To review, there are essentially three steps to rescale data to common units.

Step 1: Assign 1 to the best value in the range.
Assign 0 to the worst value in the range.

Step 2: For continuous data, assign intermediate scores proportionally:

$$\text{Scaled Score} = \frac{\left|\text{score} - \text{least preferred score}\right|}{\left|\text{most preferred score} - \text{least preferred score}\right|}$$

Step 3: For categorical data, assign intermediate scores proportionally or based on your own opinions and values.

1.2.5 Conduct an Interview to Calculate Weights

Next, Enrique and Anna assign weights to each of the measures to reflect the relative importance Enrique attaches to each of them. They decide Anna will interview Enrique. She makes observations to ensure that Enrique understands the measures he chose and the effects of the weights he assigns to each of them. They utilize Excel throughout the interview.

Anna: We have some measures and their ranges for making a decision about your college preference. Focus first on the column of least preferred values. Which one of the measures would you most want to increase from the least preferred value to its most preferred value? For example, is it more important to you to move the SAT score from 1400 to 1800 or to reduce tuition from $35,000 to $25,000?

Enrique: Lower the tuition!

Anna: Are you sure that lowering the tuition to $25,000 is the most important improvement in the whole list?

Enrique: Yes, so I think we should rank tuition number one.

Anna: Enrique, what would be the next most important measure to move from least preferred to most preferred?

Enrique: *U.S. News & World Report* ranking is important, so let's rank that second, and SAT score third.

To help them during the interview, they expanded their Excel worksheet to include columns for Rank Order, Points, and Weights (see Figure 1.2.9).

	A	B	C	D	E	F	G	H	I
1	**Measure**	**Canisius**	**Clark**	**Drexel**	**Suffolk**		**Least Preferred Value**	**Most Preferred Value**	**Rank Order**
2	**Average SAT Score**	**1590**	**1750**	**1700**	**1480**		1400	1800	3
3	Common Unit Values	0.475	0.875	0.75	0.2				
4									
5	***U.S. News & World Report* Ranking**	**2**	**1**	**1**	**3**				2
6	Common Unit Values	0.5	1	1	0.25				
7									
8	**Room & Board**	**10150**	**8850**	**12135**	**11960**		14000	8000	4
9	Common Unit Values	0.6416667	0.85833	0.31083	0.34				
10									
11	**Tuition**	**28157**	**33900**	**30470**	**25850**		35000	25000	1
12	Common Unit Values	0.6843	0.11	0.453	0.915				
13									
14	**Average Daily High Temperature**	**56**	**56**	**64**	**59**		50	70	9
15	Common Unit Values	0.3	0.3	0.7	0.45				
16									
17	**Nearness to Home**	**3**	**2**	**1**	**2**				6
18	Common Unit Values	1	0.5	0	0.5				
19									
20	**Athletics**	**1**	**3**	**1**	**3**				8
21	Common Unit Values	1	0	1	0				
22									
23	**Reputation**	**2**	**1**	**1**	**2**				7
24	Common Unit Values	1	0.5	0.5	1				
25									
26	**Size**	**2**	**1**	**4**	**2**				5
27	Common Unit Values	0.75	1	0	0.75				
28									

Figure 1.2.9: Ranking the measures

The next task is to subjectively assign points from 0 to 100 for each measure based on the rank order. The points assigned reflected the relative importance Enrique places on each measure. They continue the interview to assign these points. During this interview process, Anna encourages Enrique to think about

the relative importance of moving between the best and worst values of the two measures being considered. This is seen in the next part of the interview

Anna: Let's start by assigning 100 points to the tuition range, which you've ranked first. Now, you've ranked *U. S. News & World Report* rating second. How important is this rating, from worst to best, compared to reducing the cost of tuition from $35,000 to $25,000? If it's close, you should use a number close to 100.

Enrique: I think it's about 90% as important, so let's use 90 points for that one, and SAT scores are almost as important, so we'll use 85 points for that range.

They continue in this way. Figure 1.2.10 contains the rest of the points Enrique assigns to each of his measures as well as the sum of the points, found using the SUM function in Excel.

J29 =SUM(J2,J5,J8,J11,J14,J17,J20,J23,J26)

	A	B	C	D	E	F	G	H	I	J
1	**Measure**	**Canisius**	**Clark**	**Drexel**	**Suffolk**		**Least Preferred Value**	**Most Preferred Value**	**Rank Order**	**Points**
2	**Average SAT Score**	**1590**	**1750**	**1700**	**1480**		1400	1800	3	85
3	Common Unit Values	0.475	0.875	0.75	0.2					
4										
5	***U.S. News & World Report* Ranking**	**2**	**1**	**1**	**3**				2	90
6	Common Unit Values	0.5	1	1	0.25					
7										
8	**Room & Board**	**10150**	**8850**	**12135**	**11960**		14000	8000	4	80
9	Common Unit Values	0.6416667	0.85833	0.31083	0.34					
10										
11	**Tuition**	**28157**	**33900**	**30470**	**25850**		35000	25000	1	100
12	Common Unit Values	0.6843	0.11	0.453	0.915					
13										
14	**Average Daily High Temperature**	**56**	**56**	**64**	**59**		50	70	9	20
15	Common Unit Values	0.3	0.3	0.7	0.45					
16										
17	**Nearness to Home**	**3**	**2**	**1**	**2**				6	60
18	Common Unit Values	1	0.5	0	0.5					
19										
20	**Athletics**	**1**	**3**	**1**	**3**				8	30
21	Common Unit Values	1	0	1	0					
22										
23	**Reputation**	**2**	**1**	**1**	**2**				7	50
24	Common Unit Values	1	0.5	0.5	1					
25										
26	**Size**	**2**	**1**	**4**	**2**				5	70
27	Common Unit Values	0.75	1	0	0.75					
28										
29									**Sum:**	585

Figure 1.2.10: Assigning points to the measures

The interview continues:

Anna: Enrique, what did you get for the total number of points for all your measures? Once you have the point total, you'll need to divide the points for each measure by this total to get the weight.

Enrique: I got 585 total points. Now I can calculate the weights.

The weights Enrique calculates appear in Figure 1.2.11. These were calculated by dividing the points for a particular measure by the total points. For example, Average SAT Score has a point value of 85. So, the weight for this measure is:

$$\frac{85}{585} = 0.145 .$$

K2 =J2/J29

	A	B	C	D	E	F	G	H	I	J	K
1	Measure	Canisius	Clark	Drexel	Suffolk		Least Preferred Value	Most Preferred Value	Rank Order	Points	Weight
2	**Average SAT Score**	**1590**	**1750**	**1700**	**1480**		1400	1800	3	85	0.1452991
3	Common Unit Values	0.475	0.875	0.75	0.2						
4											
5	***U.S. News & World Report* Ranking**	**2**	**1**	**1**	**3**				2	90	0.1538462
6	Common Unit Values	0.5	1	1	0.25						
7											
8	**Room & Board**	**10150**	**8850**	**12135**	**11960**		14000	8000	4	80	0.1367521
9	Common Unit Values	0.6416667	0.85833	0.31083	0.34						
10											
11	**Tuition**	**28157**	**33900**	**30470**	**25850**		35000	25000	1	100	0.1709402
12	Common Unit Values	0.6843	0.11	0.453	0.915						
13											
14	**Average Daily High Temperature**	**56**	**56**	**64**	**59**		50	70	9	20	0.034188
15	Common Unit Values	0.3	0.3	0.7	0.45						
16											
17	**Nearness to Home**	**3**	**2**	**1**	**2**				6	60	0.1025641
18	Common Unit Values	1	0.5	0	0.5						
19											
20	**Athletics**	**1**	**3**	**1**	**3**				8	30	0.0512821
21	Common Unit Values	1	0	1	0						
22											
23	**Reputation**	**2**	**1**	**1**	**2**				7	50	0.0854701
24	Common Unit Values	1	0.5	0.5	1						
25											
26	**Size**	**2**	**1**	**4**	**2**				5	70	0.1196581
27	Common Unit Values	0.75	1	0	0.75						
28											
29									**Sum:**	585	

Figure 1.2.11: Calculating weights for each measure

Next, Anna wants to ensure that Enrique has assigned an appropriate weight to each *criterion*. The interview continues.

Anna: Enrique, what is the total weight for each criterion?

Enrique: I get a total of 0.299 for academics, 0.308 for cost, 0.137 for location, and 0.256 for social life.

Anna: Which criterion has the greatest weight assigned to it?

Enrique: It looks like cost, with 0.308.

Anna: Are there criteria with similar weights?

Enrique: It looks like academics and cost are almost the same.

Anna: Are these the criteria you feel are the most important criteria for choosing a college, and do you think they're about the same in importance?

Enrique: I didn't realize I placed so much importance on academics.

Anna: What did you expect to happen?

Enrique: I thought social life would be at the top of the list!

Anna: Well, you gave athletics only 30 points, reputation 50 points, and size 70 points. Do you want to change anything?

Enrique: No, I really think academics and cost are most important.

1.2.6 Calculate Total Scores

Next, Enrique and Anna calculate a total score for each school using the weights and the common units found in the previous sections. In his Excel worksheet, Enrique calculates the product of the weight and the corresponding common unit:

$$\text{Score} = W \cdot CU$$

For example, the common unit score for the average SAT score at Canisius College is 0.475, and the weight Enrique has assigned to average SAT score is 0.145. Multiplying these two numbers yields approximately 0.069. This value appears in cell B4 in Figure 1.2.12. The rest of the scores in Figure 1.2.12 are computed in the same way. Enrique uses absolute cell references and the Fill Handle to find the scores in Excel.

B4 =B3*K2

	A	B	C	D	E	F	G	H	I	J	K
1	**Measure**	**Canisius**	**Clark**	**Drexel**	**Suffolk**		**Least Preferred Value**	**Most Preferred Value**	**Rank Order**	**Points**	**Weight**
2	**Average SAT Score**	**1590**	**1750**	**1700**	**1480**		1400	1800	3	85	0.1452991
3	Common Unit Values	0.475	0.875	0.75	0.2						
4	Weights	0.0690171	0.12714	0.10897	0.02906						
5	***U.S. News & World Report* Ranking**	**2**	**1**	**1**	**3**				2	90	0.1538462
6	Common Unit Values	0.5	1	1	0.25						
7	Weights	0.0769231	0.15385	0.15385	0.03846						
8	**Room & Board**	**10150**	**8850**	**12135**	**11960**		14000	8000	4	80	0.1367521
9	Common Unit Values	0.6416667	0.85833	0.31083	0.34						
10	Weights	0.0877493	0.11738	0.04251	0.0465						
11	**Tuition**	**28157**	**33900**	**30470**	**25850**		35000	25000	1	100	0.1709402
12	Common Unit Values	0.6843	0.11	0.453	0.915						
13	Weights	0.1169744	0.0188	0.07744	0.15641						
14	**Average Daily High Temperature**	**56**	**56**	**64**	**59**		50	70	9	20	0.034188
15	Common Unit Values	0.3	0.3	0.7	0.45						
16	Weights	0.0102564	0.01026	0.02393	0.01538						
17	**Nearness to Home**	**3**	**2**	**1**	**2**				6	60	0.1025641
18	Common Unit Values	1	0.5	0	0.5						
19	Weights	0.1025641	0.05128	0	0.05128						
20	**Athletics**	**1**	**3**	**1**	**3**				8	30	0.0512821
21	Common Unit Values	1	0	1	0						
22	Weights	0.0512821	0	0.05128	0						
23	**Reputation**	**2**	**1**	**1**	**2**				7	50	0.0854701
24	Common Unit Values	1	0.5	0.5	1						
25	Weights	0.0854701	0.04274	0.04274	0.08547						
26	**Size**	**2**	**1**	**4**	**2**				5	70	0.1196581
27	Common Unit Values	0.75	1	0	0.75						
28	Weights	0.0897436	0.11966	0	0.08974						
29									**Sum:**	585	

Figure 1.2.12: Calculating the scores for each measure

Then, Enrique used the SUM function in Excel to find the total scores for each college that appear in row 20 in Figure 1.2.13. Now Enrique can see which of his college choices best suits his preferences.

B29 =SUM(B4,B7,B10,B13,B16,B19,B22,B25,B28)

	A	B	C	D	E	F	G	H	I	J	K
1	**Measure**	**Canisius**	**Clark**	**Drexel**	**Suffolk**		**Least Preferred Value**	**Most Preferred Value**	**Rank Order**	**Points**	**Weight**
2	**Average SAT Score**	**1590**	**1750**	**1700**	**1480**		1400	1800	3	85	0.1452991
3	Common Unit Values	0.475	0.875	0.75	0.2						
4	Weights	0.0690171	0.12714	0.10897	0.02906						
5	***U.S. News & World Report* Ranking**	**2**	**1**	**1**	**3**				2	90	0.1538462
6	Common Unit Values	0.5	1	1	0.25						
7	Weights	0.0769231	0.15385	0.15385	0.03846						
8	**Room & Board**	**10150**	**8850**	**12135**	**11960**		14000	8000	4	80	0.1367521
9	Common Unit Values	0.6416667	0.85833	0.31083	0.34						
10	Weights	0.0877493	0.11738	0.04251	0.0465						
11	**Tuition**	**28157**	**33900**	**30470**	**25850**		35000	25000	1	100	0.1709402
12	Common Unit Values	0.6843	0.11	0.453	0.915						
13	Weights	0.1169744	0.0188	0.07744	0.15641						
14	**Average Daily High Temperature**	**56**	**56**	**64**	**59**		50	70	9	20	0.034188
15	Common Unit Values	0.3	0.3	0.7	0.45						
16	Weights	0.0102564	0.01026	0.02393	0.01538						
17	**Nearness to Home**	**3**	**2**	**1**	**2**				6	60	0.1025641
18	Common Unit Values	1	0.5	0	0.5						
19	Weights	0.1025641	0.05128	0	0.05128						
20	**Athletics**	**1**	**3**	**1**	**3**				8	30	0.0512821
21	Common Unit Values	1	0	1	0						
22	Weights	0.0512821	0	0.05128	0						
23	**Reputation**	**2**	**1**	**1**	**2**				7	50	0.0854701
24	Common Unit Values	1	0.5	0.5	1						
25	Weights	0.0854701	0.04274	0.04274	0.08547						
26	**Size**	**2**	**1**	**4**	**2**				5	70	0.1196581
27	Common Unit Values	0.75	1	0	0.75						
28	Weights	0.0897436	0.11966	0	0.08974						
29	**Total Score**	0.6899801	0.6411	0.50071	0.51231				**Sum:**	585	

Figure 1.2.13: Calculating the total scores of Enrique's schools

1.2.7 Interpret Results

Enrique reviews these results carefully. He notices that Drexel and Suffolk have scored much lower than his top-ranked choice, so he excludes them from further study. However, he decides to take a closer look at the relative strengths and weaknesses of Canisius, ranked first, and Clark, ranked second. There is only a 0.049 difference between the two, and he is not sure that it is enough evidence to make this critical life decision.

Q12. What are some reasons why Enrique may not choose Canisius, even though it was ranked first?

Q13. On many of the measures, Clark received better scores than Canisius. Why did Canisius end up having the higher total score?

Q14. Suppose Enrique was offered a scholarship at Clark for $5,000. In your Excel worksheet, change the tuition cost from $33,900 to $28,900 for Clark. What is the new total score for Clark? Which school would you recommend for Enrique?

Q15. Enrique considered applying for scholarships at Canisius as well. How large should the scholarship be for Canisius to have a higher score than Clark (assuming that the tuition at Clark is now $28,900)?

